The Theory of Wave Field Synthesis Revisited

Sascha Spors, Rudolf Rabenstein, and Jens Ahrens

124th Convention of the Audio Engineering Society
Amsterdam, 2008 May 17–20

Deutsche Telekom Laboratories

Deutsche Telekom AG
Laboratories

Motivation

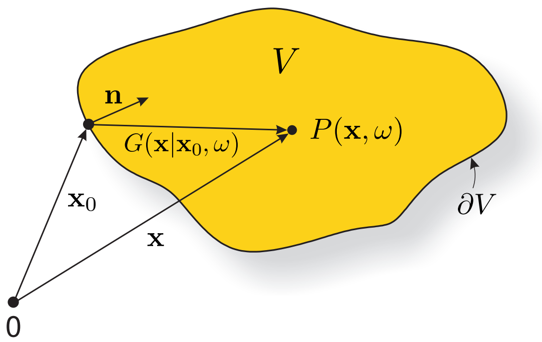
- Wave Field Synthesis (WFS) has been developed almost 20 years ago
- the theory is well documented for linear loudspeaker arrays
- some extensions towards two/three dimensional arbitrarily shaped WFS systems
- curved/closed-contour arrays are often used in practice

This paper: Revisits the theoretical foundations of WFS and presents a generalized theory of WFS.

Fundamentals of Sound Field Reproduction

The Kirchhoff-Helmholtz integral provides the solution of the homogeneous wave equation with respect to inhomogeneous boundary conditions

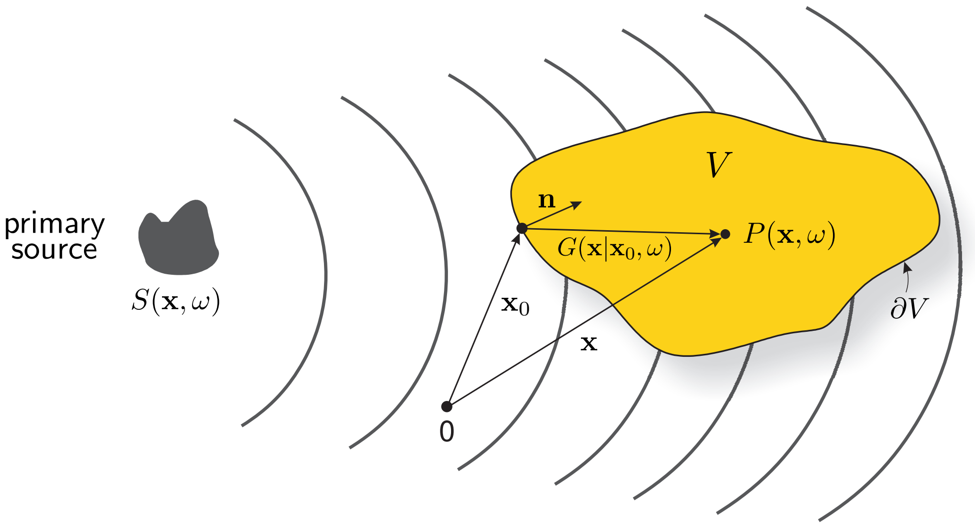
$$P(\mathbf{x},\omega) = -\oint_{\partial V} \left(G(\mathbf{x}|\mathbf{x}_0,\omega)\,\frac{\partial}{\partial \mathbf{n}} P(\mathbf{x}_0,\omega) - P(\mathbf{x}_0,\omega)\,\frac{\partial}{\partial \mathbf{n}} G(\mathbf{x}|\mathbf{x}_0,\omega) \right)\, dS_0$$



Fundamentals of Sound Field Reproduction

The field of a primary source $S(\mathbf{x},\omega)$ within the area V is uniquely given by its pressure and pressure gradient on the boundary ∂V

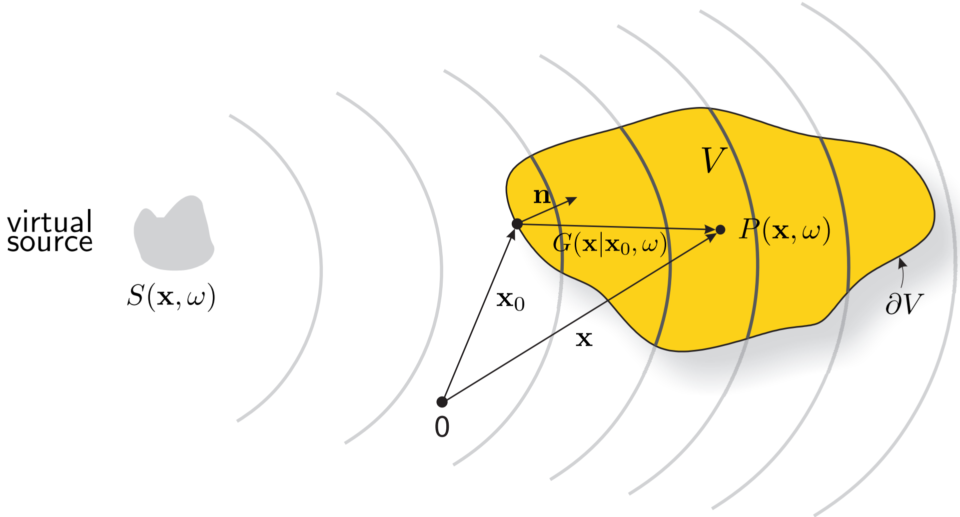
$$P(\mathbf{x},\omega) = -\oint_{\partial V} \left(G(\mathbf{x}|\mathbf{x}_0,\omega)\,\frac{\partial}{\partial \mathbf{n}} S(\mathbf{x}_0,\omega) - S(\mathbf{x}_0,\omega)\,\frac{\partial}{\partial \mathbf{n}} G(\mathbf{x}|\mathbf{x}_0,\omega) \right)\, dS_0$$



Fundamentals of Sound Field Reproduction

The Green's function and its gradient can be interpreted as sources (secondary sources) that generate the field of a virtual source $S(\mathbf{x},\omega)$ inside the listening area V

$$P(\mathbf{x},\omega) = -\oint_{\partial V} \left(G(\mathbf{x}|\mathbf{x}_0,\omega)\,\frac{\partial}{\partial \mathbf{n}} S(\mathbf{x}_0,\omega) - S(\mathbf{x}_0,\omega)\,\frac{\partial}{\partial \mathbf{n}} G(\mathbf{x}|\mathbf{x}_0,\omega) \right) \, dS_0$$



Fundamentals of Sound Field Reproduction

The theoretical basis of sound field reproduction is given by the Kirchhoff-Helmholtz integral

$$P(\mathbf{x},\omega) = -\oint_{\partial V} \left(G(\mathbf{x}|\mathbf{x}_0,\omega)\,\frac{\partial}{\partial \mathbf{n}} S(\mathbf{x}_0,\omega) - S(\mathbf{x}_0,\omega)\,\frac{\partial}{\partial \mathbf{n}} G(\mathbf{x}|\mathbf{x}_0,\omega) \right) \, dS_0$$

The explicit form of the Green's function depends on the dimensionality of the problem:

Reproduction in a volume (3D)

$$G_{0,\text{3D}}(\mathbf{x}|\mathbf{x}_0,\omega) = \frac{1}{4\pi}\frac{e^{-j\frac{\omega}{c}|\mathbf{x}-\mathbf{x}_0|}}{|\mathbf{x}-\mathbf{x}_0|}$$

$\Rightarrow$ secondary point sources

Reproduction in a plane (2D)

$$G_{0,\text{2D}}(\mathbf{x}|\mathbf{x}_0,\omega) = \frac{j}{4} H_0^{(2)}\left(\frac{\omega}{c}\,|\mathbf{x}-\mathbf{x}_0|\right)$$

$\Rightarrow$ secondary line sources

$G(\mathbf{x}|\mathbf{x}_0,\omega) \quad \Rightarrow$ **monopole secondary source**

$\frac{\partial}{\partial \mathbf{n}} G(\mathbf{x}|\mathbf{x}_0,\omega) \quad \Rightarrow$ **dipole secondary source**

Elimination of Dipole Secondary Sources

Elimination of dipole secondary sources desirable

- halves the number of required secondary sources
- monopole sources can be realized quite well by loudspeakers

There exist two different approaches to eliminate the dipole secondary sources*:

1. **The 'Simple Source Approach'**
   - provides formulation for monopole only reproduction
   - solution not unique
   - basis of higher-order Ambisonics (HOA)
2. **Modification of Green's function used in the Kirchhoff-Helmholtz integral**

*according to [E.G. Williams, *Fourier Acoustics*, 1999]

Neumann Green's Function

Concept: Impose Neumann boundary condition to $G(\mathbf{x}|\mathbf{x}_0, \omega)$ on secondary source contour ∂V to eliminate the dipole secondary sources.

Neumann Green's function

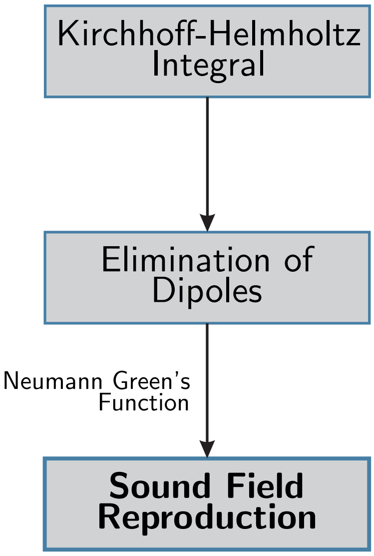
$$\frac{\partial}{\partial \mathbf{n}} G_{\mathsf{N}}(\mathbf{x}|\mathbf{x}_0, \omega)\Big|_{\mathbf{x}_0 \in \partial V} = 0$$

Reproduced wave field

$$P(\mathbf{x}, \omega) = -\oint_{\partial V} \frac{\partial}{\partial \mathbf{n}} S(\mathbf{x_0}, \omega)\; G_{\mathsf{N}}(\mathbf{x}|\mathbf{x}_0, \omega)\; dS_0$$

- $G_{\mathsf{N}}(\mathbf{x}|\mathbf{x}_0, \omega)$ may be hard to find for complex geometries
- $G_{\mathsf{N}}(\mathbf{x}|\mathbf{x}_0, \omega)$ may be impossible to realize by existing secondary sources
- resulting wave field equivalent to simple source formulation

Overview: Theory of Sound Field Reproduction/WFS

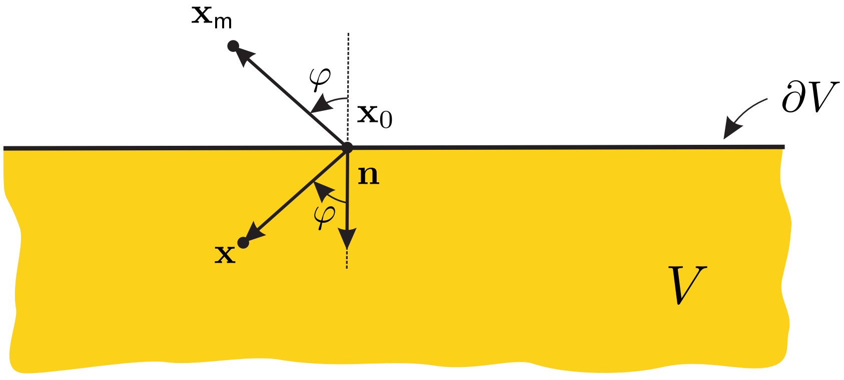


- exact for all geometries

Linear/Planar Secondary Source Contours

Neumann Green's function for a linear/planar secondary source contour ∂V

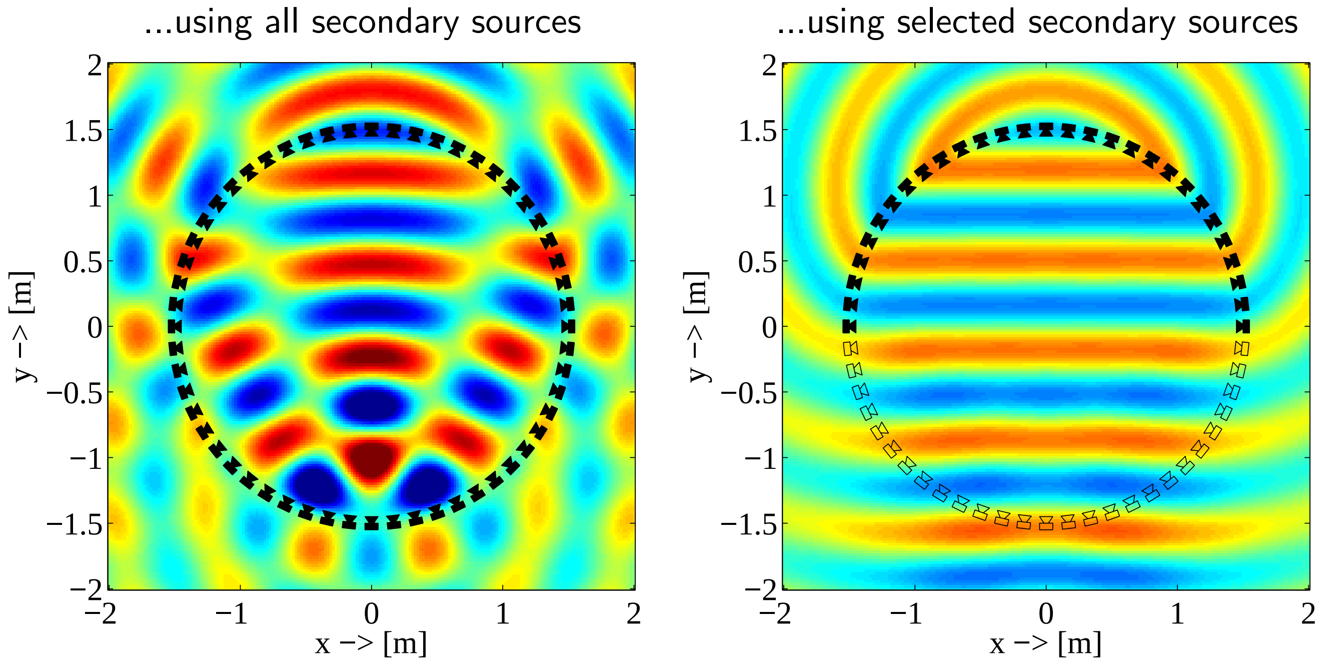
$$G_{\mathsf{N}}(\mathbf{x}|\mathbf{x}_0,\omega) = 2\ G_0(\mathbf{x}|\mathbf{x}_0,\omega)$$



- secondary sources are monopoles
- provides exact reproduction for (infinite) linear/planar contours
- theoretical basis of Rayleigh I integral

Example: Selection of Active Secondary Sources

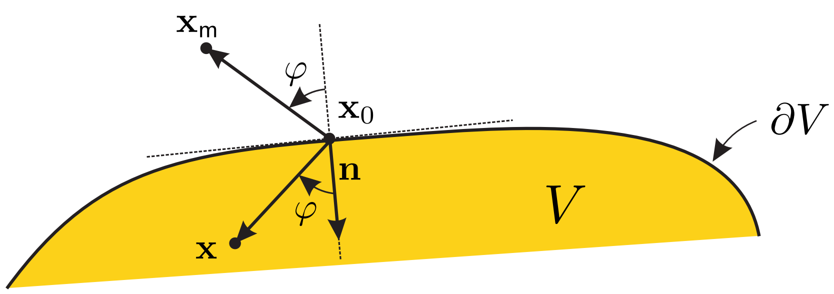
Reproduction of monochromatic plane wave with two-dimensional WFS...



[$R = 1.50$ m, $N = 56$, $f_{\text{pw}} = 500$ Hz, $\alpha_{\text{pw}} = 90^{\circ}$]

Curved/Closed Secondary Source Contours

The Neumann Green's function for a linear/planar secondary source contour is typically used also for curved contours ∂V in WFS



Potential artefacts

1. Neumann boundary condition not fulfilled on curved ∂V

   $\Rightarrow$ **undesired contributions in reproduced wave field**

2. wavefield outside V may reenter listening area V

   $\Rightarrow$ boundary ∂V has to be convex

Secondary Source Selection

Introduction of window function to eliminate undesired contributions

$$P(\mathbf{x},\omega) = -\oint_{\partial V} 2\, a(\mathbf{x}_0)\, \frac{\partial}{\partial \mathbf{n}} S(\mathbf{x_0},\omega)\, G_0(\mathbf{x}|\mathbf{x}_0,\omega)\, dS_0$$

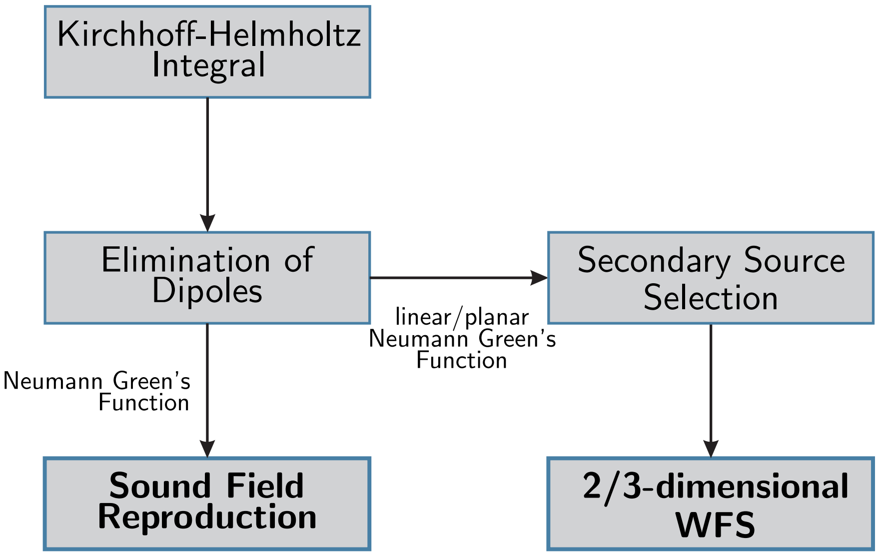
Intuitive definition of window function

$$a(\mathbf{x}_0) = \begin{cases} 1 & \text{, if the local propagation direction of } S(\mathbf{x_0},\omega) \text{ coincides with } \mathbf{n}, \\ 0 & \text{, otherwise.} \end{cases}$$

- analytic formulation of window function using the acoustic intensity vector*
- secondary source selection is used in state of the art WFS implementations

*[Spors, 123th AES Convention, 2007]

Overview: Theory of Sound Field Reproduction/WFS

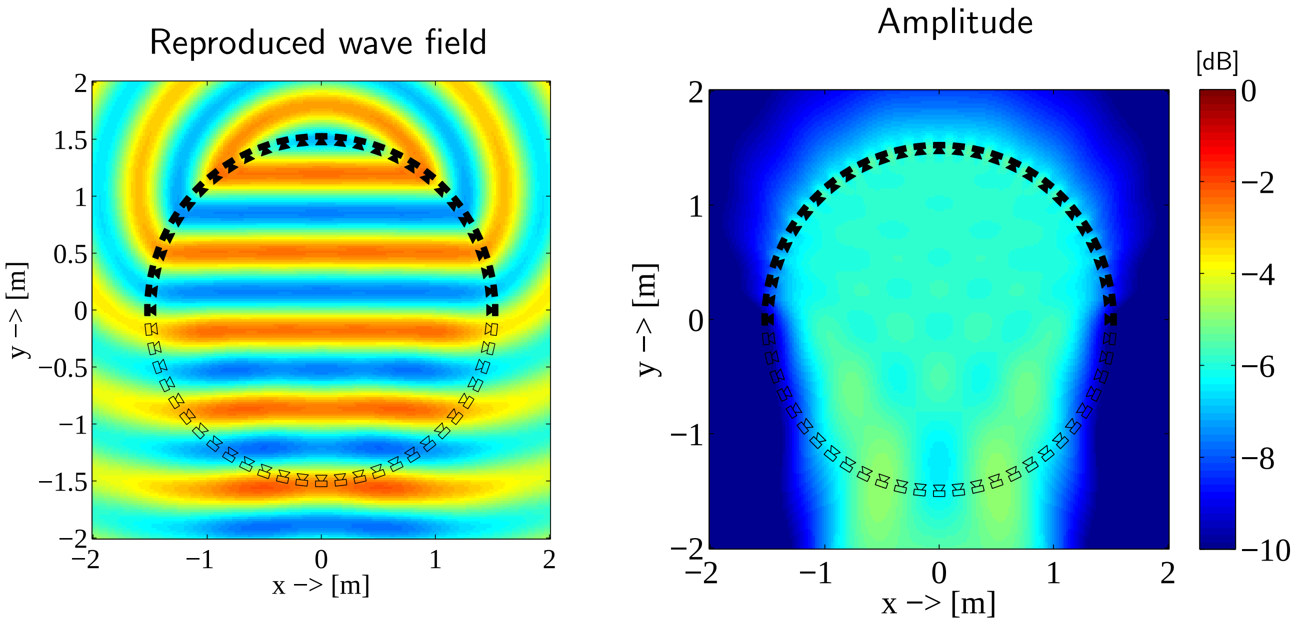


- exact for all geometries

- not exact for non-linear geometry
- closed form driving functions

Example: Artifacts of linear Neumann Green's Function

Reproduction of monochromatic plane wave with two-dimensional WFS



[$R = 1.50$ m, $N = 56$, $f_{\text{pw}} = 500$ Hz, $\alpha_{\text{pw}} = 90°$]

Secondary Source Type Mismatch

Typical systems aim at correct reproduction in a **plane** using secondary **point sources**

⇒ **Mismatch of secondary source types!**

The large-argument (far-field) approximation of the Hankel function

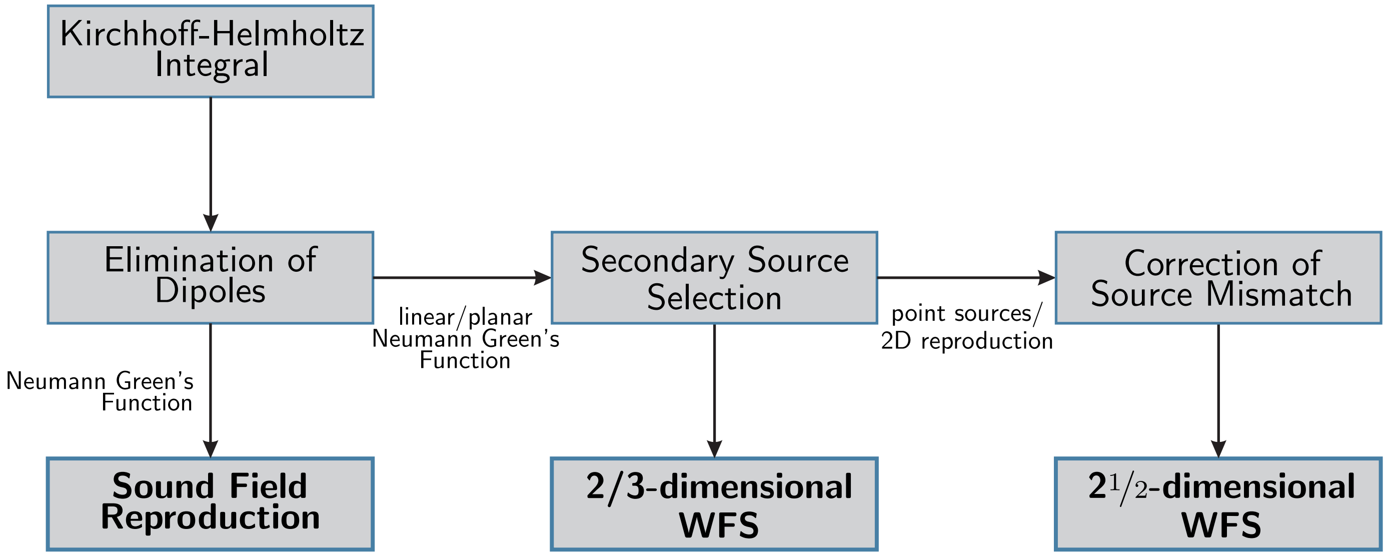
$$G_{0,\text{2D}}(\mathbf{x}|\mathbf{x}_0,\omega) \approx \sqrt{\frac{2\pi}{jk}}\,\sqrt{|\mathbf{x}-\mathbf{x}_0|}\;\;\frac{e^{-jk|\mathbf{x}-\mathbf{x}_0|}}{4\pi\,|\mathbf{x}-\mathbf{x}_0|}$$

states that secondary line sources can be exchanged by point sources when applying

- a geometry independent spectral correction, and
- an amplitude correction with respect to a reference position/line

Theoretical basis of almost all current implementations of WFS
⇒ here termed as 2½-dimensional WFS

Overview: Theory of Sound Field Reproduction/WFS



- exact for all geometries

- not exact for non-linear geometry
- closed form driving functions

- basis of current WFS
- amplitude errors
- spectral errors

Unified Driving Function for WFS

$$D(\mathbf{x}_0,\omega) = 2a(\mathbf{x}_0)\, w(\mathbf{x}_0) f(\omega)\, \frac{\partial}{\partial \mathbf{n}} S(\mathbf{x_0},\omega)$$

- $a(\mathbf{x}_0)$: window function for secondary source selection
- $S(\mathbf{x_0},\omega)$: wave field of desired virtual source
- $w(\mathbf{x}_0)$: amplitude correction for 2½D WFS (if required)
- $f(\omega)$: spectral correction for 2½D WFS (if required)

Derivation of explicit driving functions (in paper) for:

- spherical and plane waves as wave fields for the virtual sources
- two- and three-dimensional WFS
- 2½-dimensional WFS

Artifacts of WFS

Artifacts of two/three-dimensional WFS

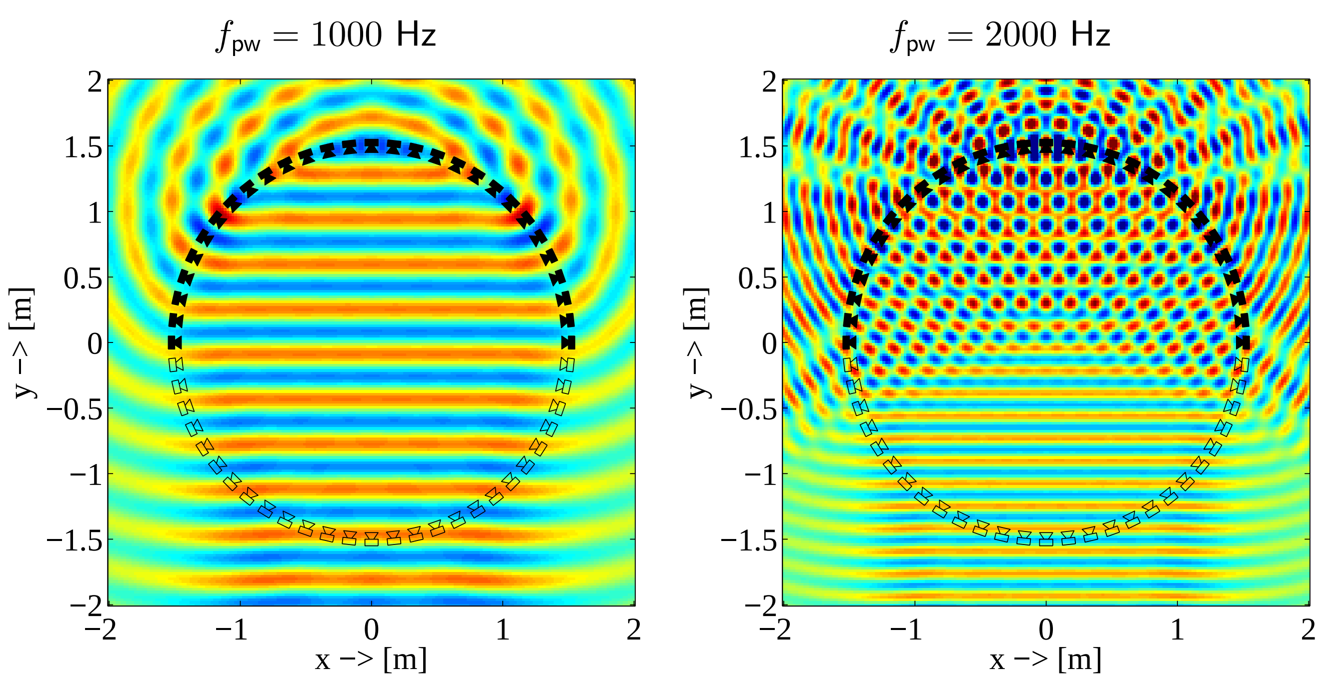
- spatial sampling of secondary source distribution
  $\Rightarrow$ may lead to spatial aliasing artifacts
- truncation of secondary source distribution
  $\Rightarrow$ may lead to truncation artifacts
- reproduction of moving virtual sources
  $\Rightarrow$ may lead to various artifacts

Additional artifacts of 2$^1/_2$-dimensional WFS

- secondary source type mismatch
  $\Rightarrow$ amplitude errors
- out of reproduction plane listeners
  $\Rightarrow$ amplitude errors, localization errors

Example: Spatial Sampling Artifacts of WFS

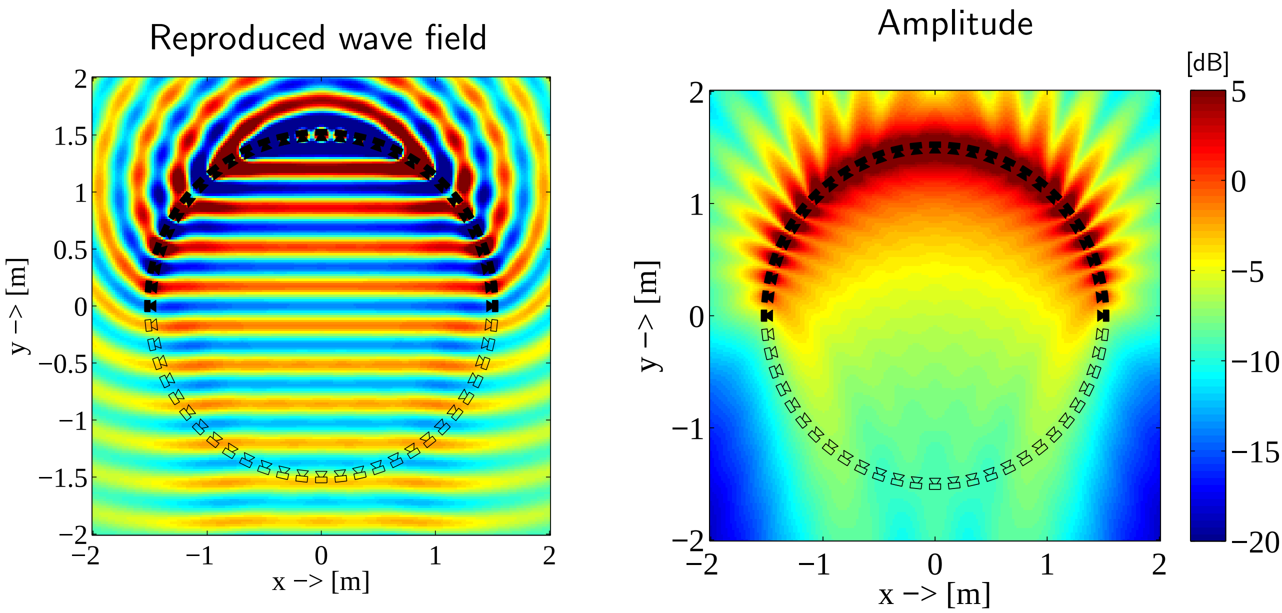
Reproduction of monochromatic plane wave with two-dimensional WFS



[$R = 1.50$ m, $N = 56$, $\alpha_{pw} = 90°$]

Example: Amplitude Errors of 2½D WFS

Reproduction of monochromatic plane wave with 2½-dimensional WFS



$[R = 1.50 \text{ m}, N = 56, f_{\text{pw}} = 1000 \text{ Hz}, \alpha_{\text{pw}} = 90°]$

Summary

This paper revisits the foundations of Wave Field Synthesis and presents a generalized theory covering various known and novel aspects.

- theoretical framework for 2D/3D and 2½D reproduction
- analytic driving functions for spherical and plane waves

Artifacts of WFS:

- no exact reproduction for non linear/planar secondary source contours
- spatial aliasing
- amplitude errors (2½D WFS)

Further work:

- documentation of temporal/spatial discretization
- links to other reproduction systems (e.g. HOA)