Spatial Sampling Artifacts of Wave Field Synthesis for the Reproduction of Virtual Point Sources

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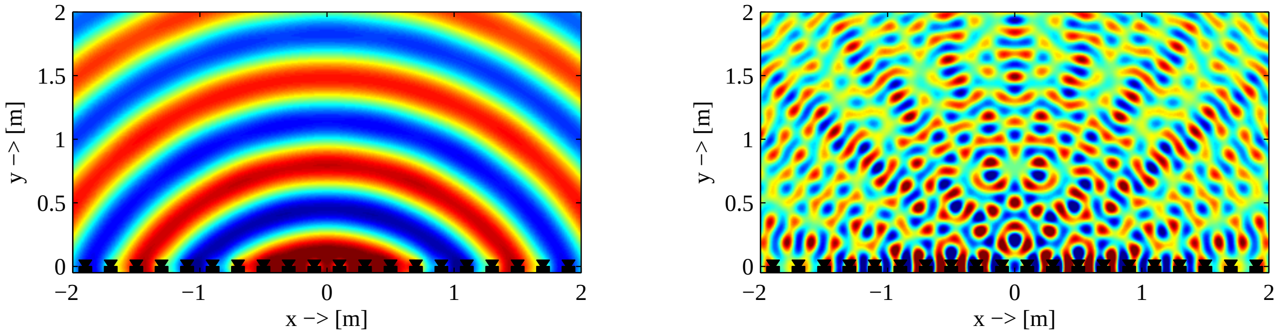
Motivation

$f_S = 500$ Hz $f_S = 2000$ Hz



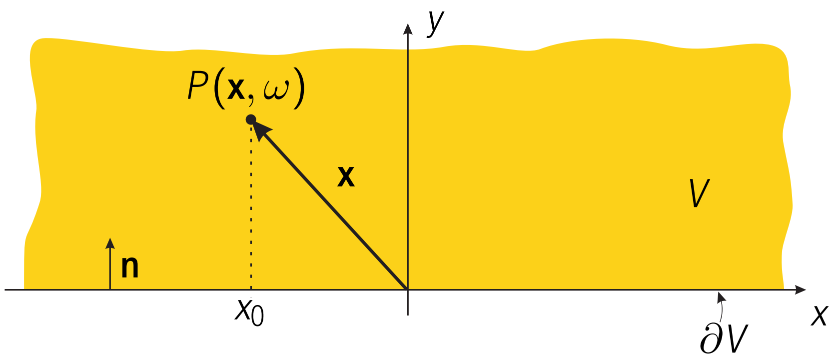
$[\mathbf{x}_S = [0 \quad -1]$ m, $\Delta x = 0.20$ m$]$

- wave field synthesis (WFS) suffers from spatial aliasing in practical implementations
- spatial aliasing artifacts for plane waves are well understood
- virtual point sources are frequently used in WFS
- here: detailed analysis of spatial aliasing artifacts for virtual point sources

Wave Field Synthesis for Linear Arrays

Rayleigh's first integral formula provides the pressure in one half space V given the directional pressure gradient on the boundary ∂V of that half space

$$P(\mathbf{x},\omega) = -2 \int_{-\infty}^{\infty} \frac{\partial}{\partial \mathbf{n}} P(\mathbf{x}_0,\omega)\, G_{0,2D}(\mathbf{x}-\mathbf{x}_0,\omega)\, dx_0$$



$\mathbf{x} = [x\ y]^T$

$\mathbf{x}_0 = [x_0\ 0]^T$

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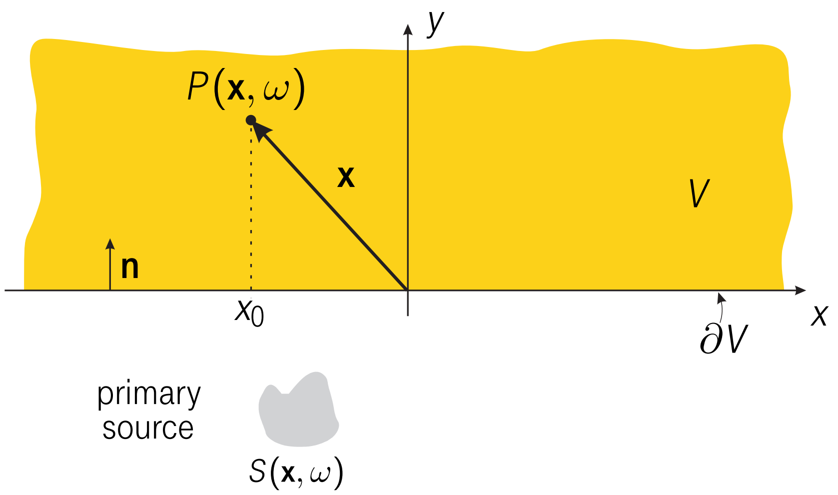
Wave Field Synthesis for Linear Arrays

The field of a (primary) source $S(\mathbf{x},\omega)$ within the area V is uniquely given by its pressure gradient on the boundary ∂V

$$P(\mathbf{x},\omega) = -2 \int_{-\infty}^{\infty} \frac{\partial}{\partial \mathbf{n}} S(\mathbf{x}_0,\omega)\, G_{0,2D}(\mathbf{x}-\mathbf{x}_0,\omega)\, dx_0$$



$\mathbf{x} = [x\ y]^T$

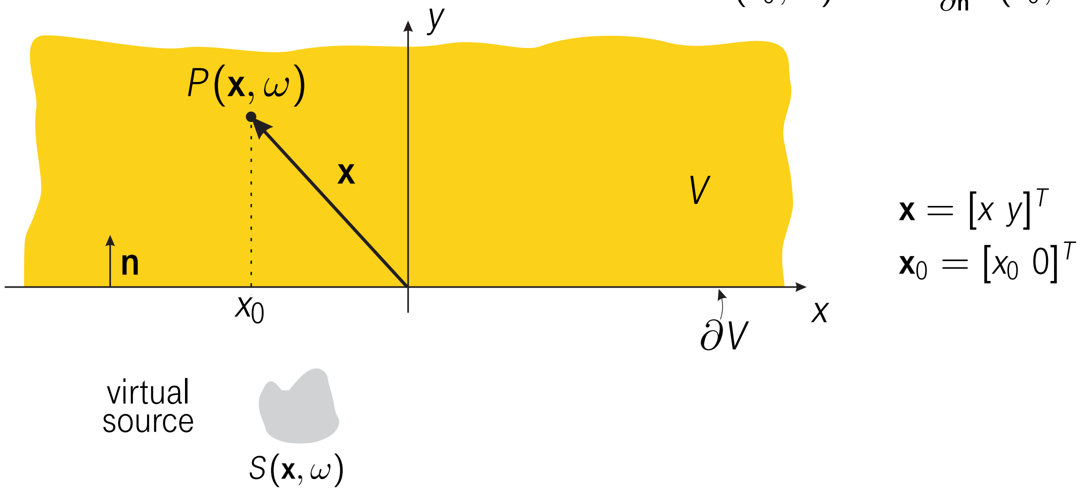
$\mathbf{x}_0 = [x_0\ 0]^T$

Wave Field Synthesis for Linear Arrays

The Green's function can be interpreted as (secondary) source that generates the field of a desired virtual source $S(\mathbf{x}, \omega)$ inside the listening area V.

$$P(\mathbf{x}, \omega) = \int_{-\infty}^{\infty} D(x_0, \omega)\, G_{0,2D}(\mathbf{x} - \mathbf{x}_0, \omega)\; dx_0$$

$$D(x_0, \omega) = -2 \frac{\partial}{\partial \mathbf{n}} S(x_0, \omega)$$



Secondary Source Types

The explicit form of the Green's function depends on the dimensionality

- two-dimensional reproduction $\longrightarrow$ secondary line sources
- three-dimensional reproduction $\longrightarrow$ secondary point sources

WFS uses secondary point sources for two-dimensional reproduction

- mismatch of secondary source type
- results in amplitude and spectral errors
- artifacts have no influence on sampling theorems (...but on near-field effects)

$\Rightarrow$ **analysis of sampling artifacts based on theory of two-dimensional WFS**

Spatio-temporal Frequency Domain Description

Reproduced wave field using a linear distribution of secondary sources

$$P(\mathbf{x},\omega) = \int_{-\infty}^{\infty} D(x_0,\omega)\, G_0(\mathbf{x}-\mathbf{x}_0,\omega)\; dx_0$$

Spatial Fourier transformation $\mathcal{F}_x$

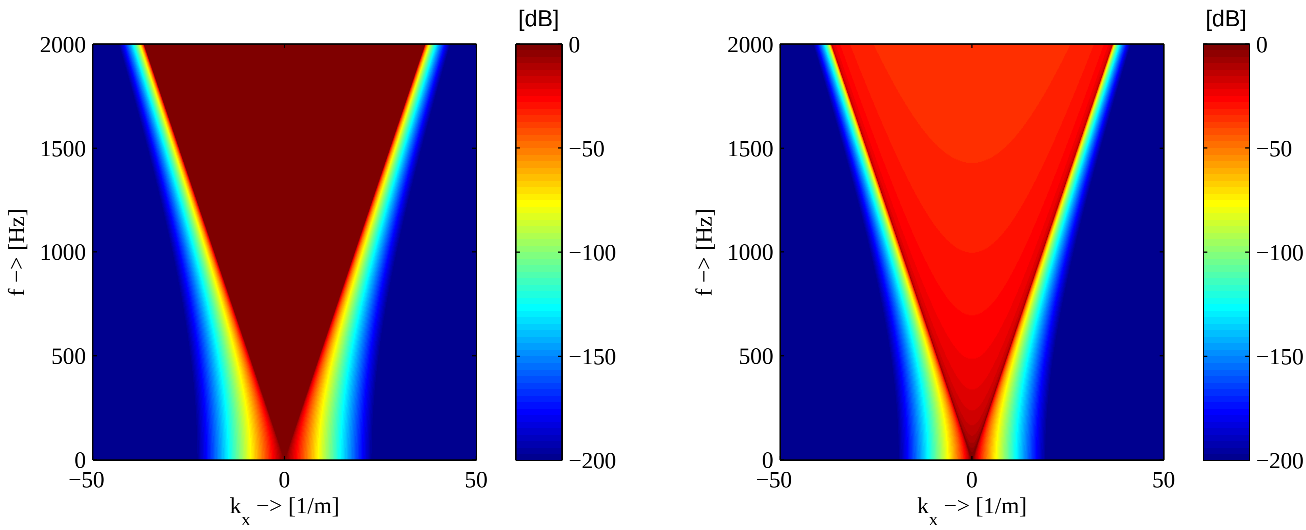
$$\mathbf{x} = [x\ y]^T$$
$$\mathbf{x}_0 = [x_0\ 0]^T$$

$$\tilde{P}(k_x,y,\omega) = \tilde{D}(k_x,\omega)\ \cdot\ \tilde{G}_0(k_x,y,\omega)$$

- spatio-temporal frequency domain description of reproduced wave field
- provides insights into structure of spatial aliasing
- allows to identify the spatial aliasing components

Example: Spectrum of Driving Function / Secondary Sources

driving function **secondary source**

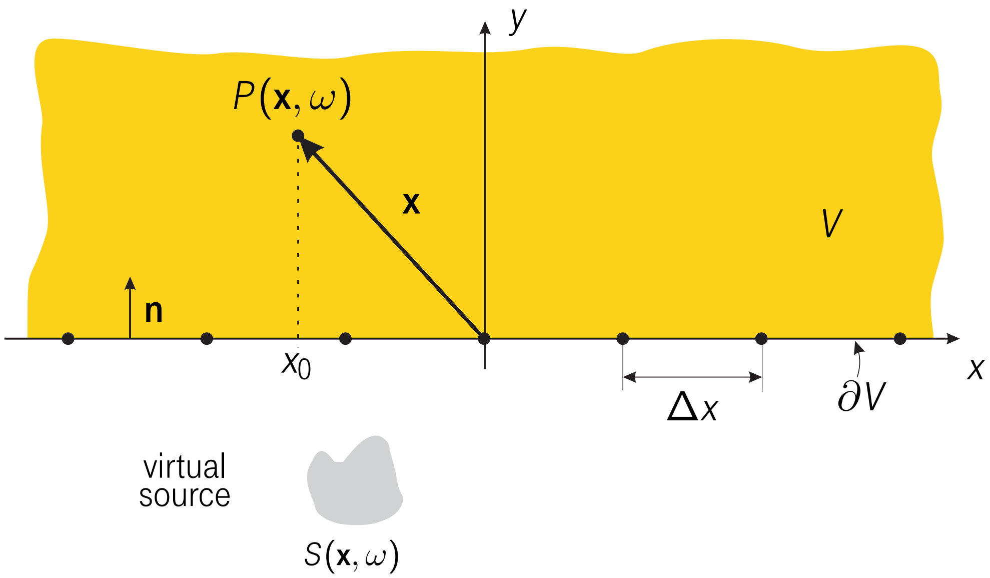


$[\mathbf{x}_s = [0\ \ -1]\ \mathrm{m},\ y = 1\ \mathrm{m}]$

- propagating for $|k_x| \leq \left|\frac{\omega}{c}\right|$, evanescent for $|k_x| > \left|\frac{\omega}{c}\right|$, $\omega = 2\pi f$

Sampling of Secondary Source Distribution

Spatial sampling of continuous secondary source distribution



Δx: spatial sampling interval

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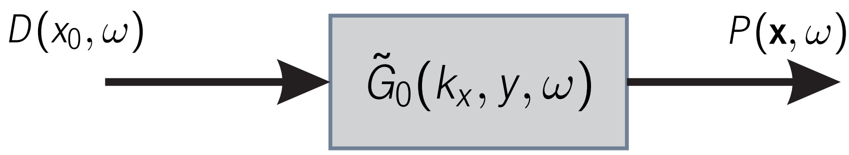
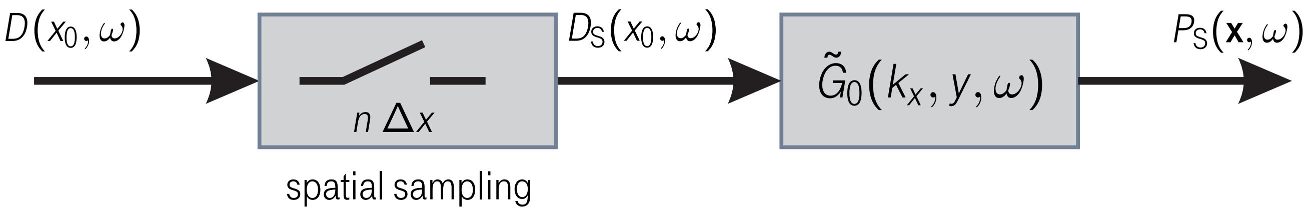
Sampling of Secondary Source Distribution

Continuous distribution of secondary sources



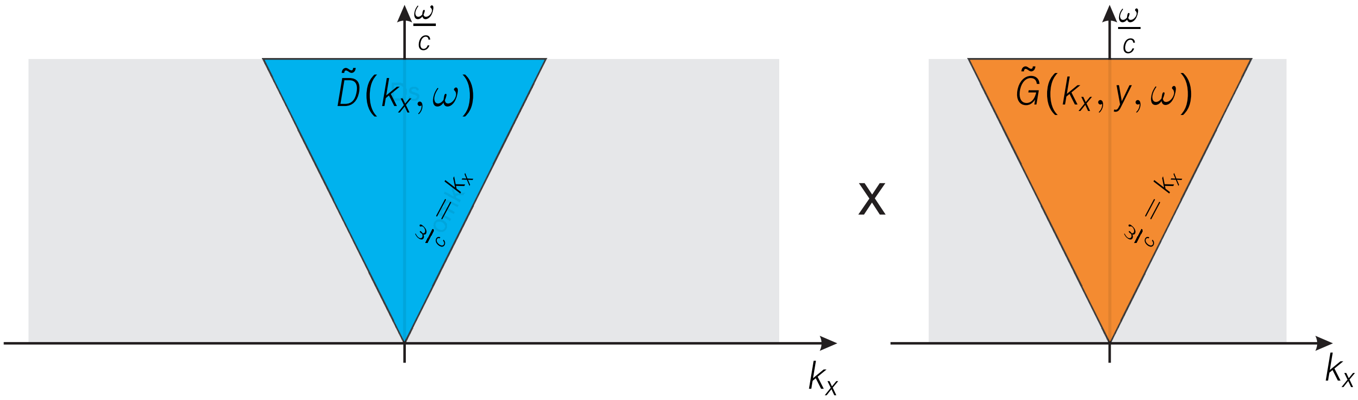
Spatially discrete distribution of secondary sources



- sampling leads to repetitions of spectrum of driving function
- weighted by secondary source spectrum (analog to interpolation)

Spectrum of Reproduced Wave Field

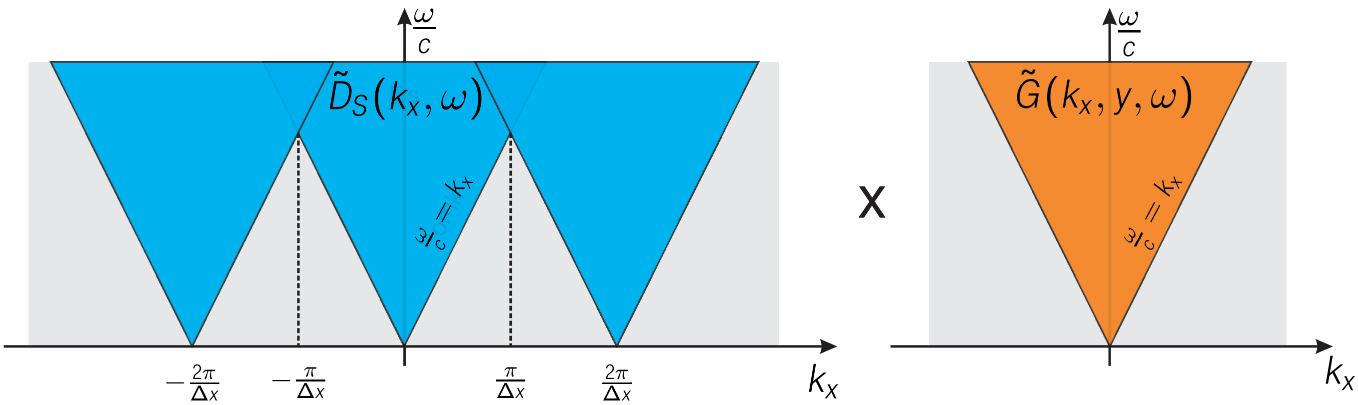
Qualitative illustration of spectra (magnitude) of driving function and secondary sources



- exact reproduction of desired wave field without sampling
- spectra not strictly bandlimited in k_x for given frequency ω
- spectral repetitions due to spatial sampling
- mixtures of propagating/evanescent contributions

Spectrum of Reproduced Wave Field

Qualitative illustration of spectra (magnitude) of driving function and secondary sources



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Theory of Secondary Source Sampling

Artifacts due to the secondary source sampling can only be avoided when

- the spectrum of the driving function is band-limited, and
- the spectrum of the secondary sources is band-limited.

Consequences of missing band-limitation

- spectral overlaps in the driving function $\longrightarrow$ aliasing
- repetitions weighted by secondary source spectrum $\longrightarrow$ reconstruction error

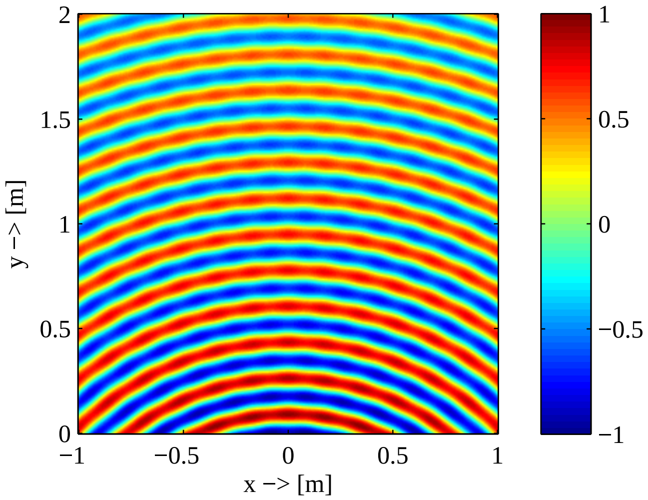
Anti-aliasing condition (considering the propagating parts only)

$$f_{\mathrm{al}} \leq \frac{c}{2\Delta x}$$

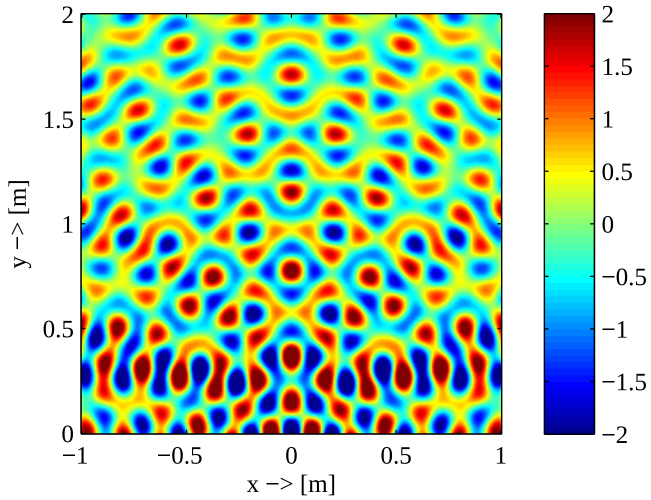
Example: $\Delta x = 0.20$ m $\Rightarrow f_{\mathrm{al}} \approx 850$ Hz

Example: Sampling Artifacts

without sampling (baseband)



sampling artifacts (spectral repetitions)



$[\mathbf{x}_s = [0 \ \ -1]$ m, $f_s = 2000$ Hz, $\Delta x = 0.20$ m$]$

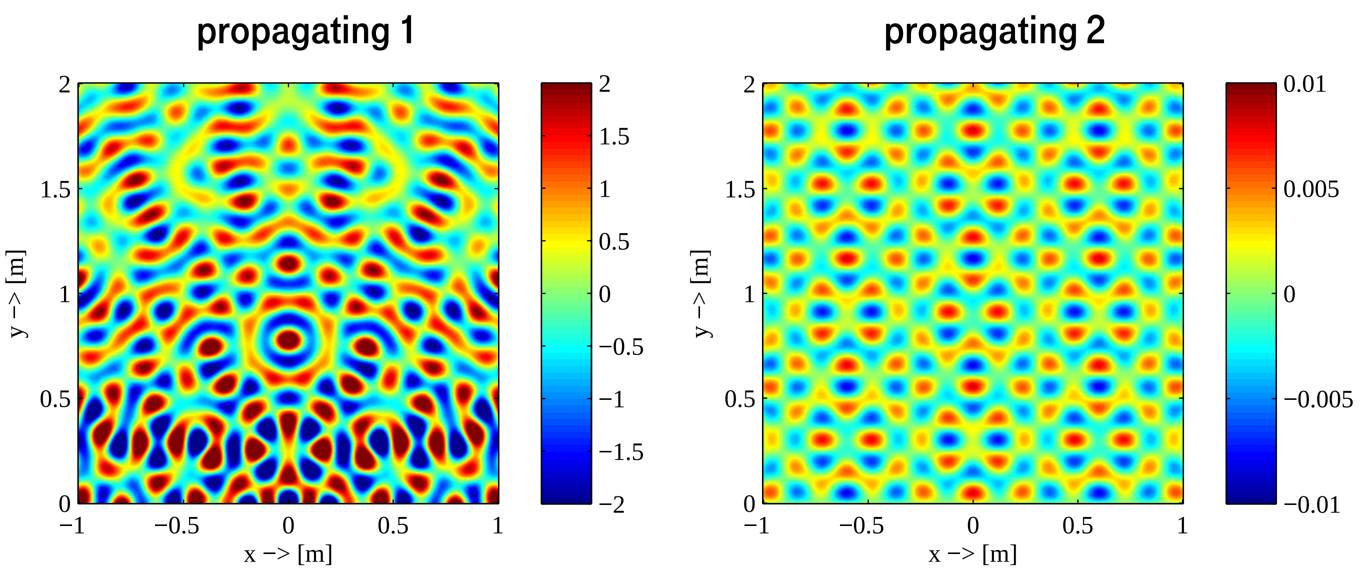
Contributions to Reproduced Wave Field

Four contributions to the reproduced spectrum can be identified

		Driving Function	
		Propagating	Evanescent
Secondary Source	Propagating	Propagating 1	Propagating 2
	Evanescent	Evanescent 1	Evanescent 2

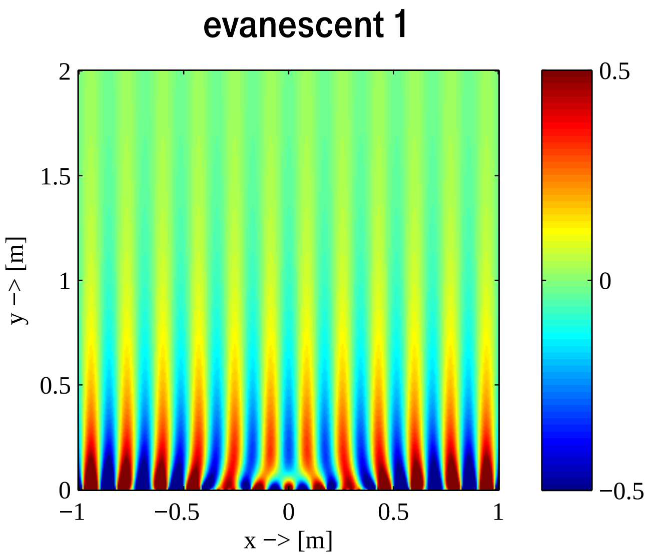
- evanescent contributions decay rapidly with distance to secondary sources
- evanescent contributions and propagating 2 occur due to spatial sampling
- perceptual relevance of evanescent contributions unclear
- propagating 1 is main contribution

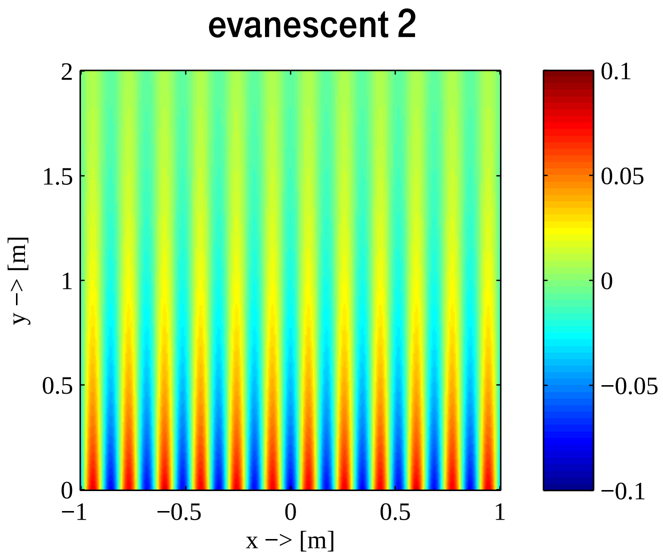
Example: Contributions to Reproduced Wave Field



[$\mathbf{x}_s = [0 \;\; -1]$ m, $f_s = 2000$ Hz, $\Delta x = 0.20$ m]

Example: Contributions to Reproduced Wave Field

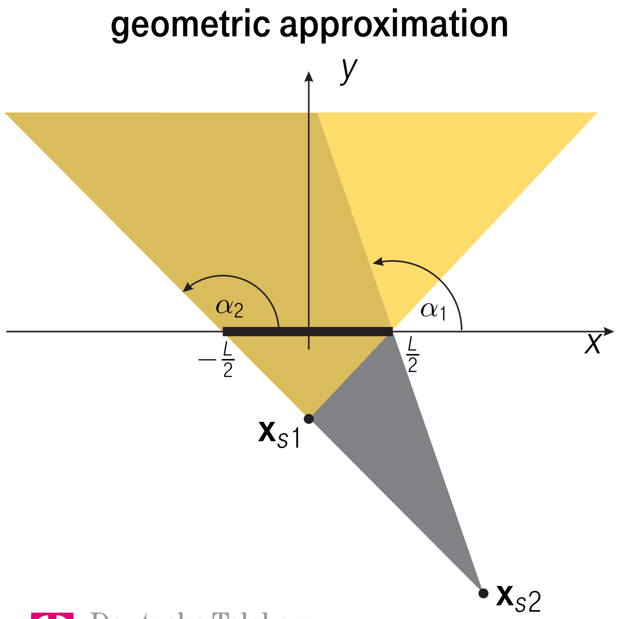


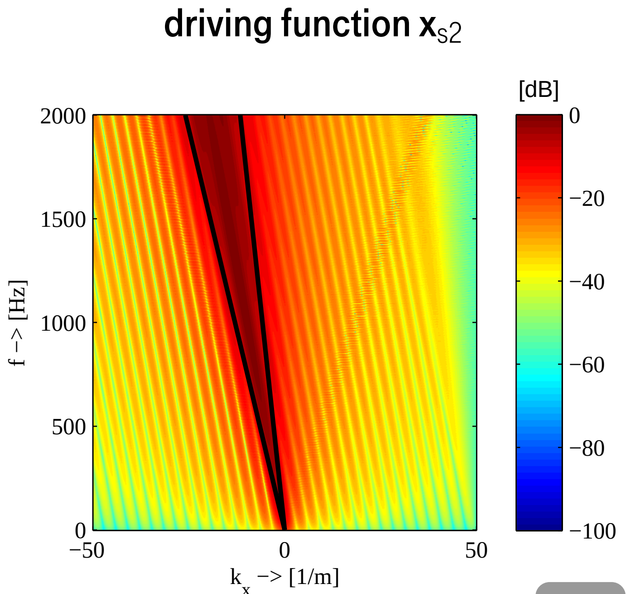


$[\mathbf{x}_s = [0 \;\; -1]$ m, $f_s = 2000$ Hz, $\Delta x = 0.20$ m]

Truncation of Secondary Source Distribution

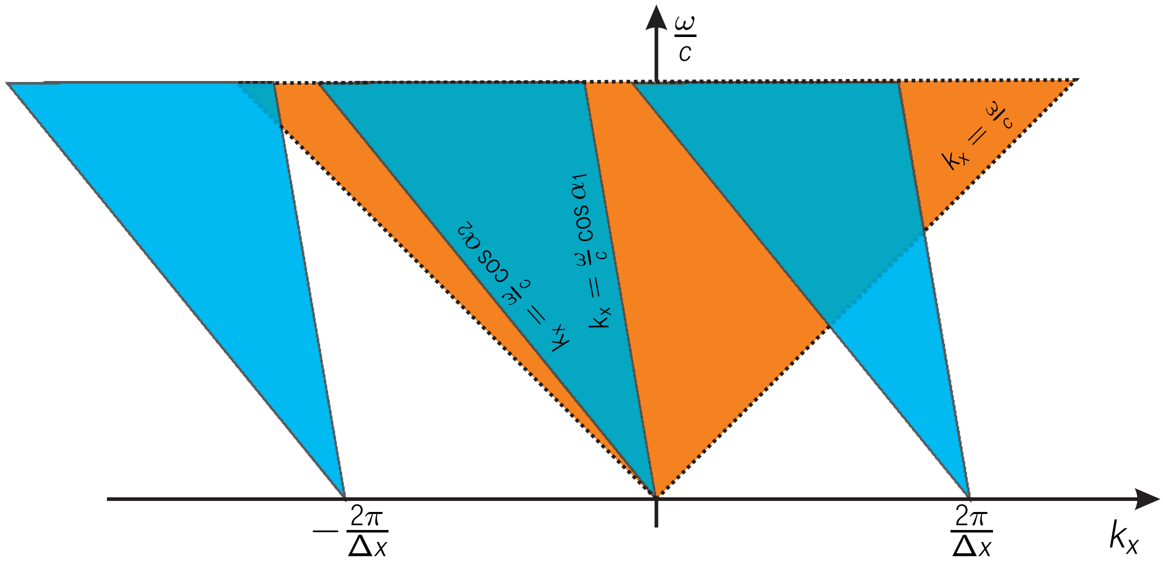
- model truncation by spatial windowing of driving function
- convolution with transformed window function in spatial frequency domain
- truncation artifacts (bending waves)





Spectrum of Reproduced Wave Field for Truncated Array

Qualitative illustration of spectra (magnitude) of driving function and secondary sources



- aliasing frequency $f_{\mathrm{tr,al1}} \geq f_{\mathrm{al}}$ higher compared to infinitely long array

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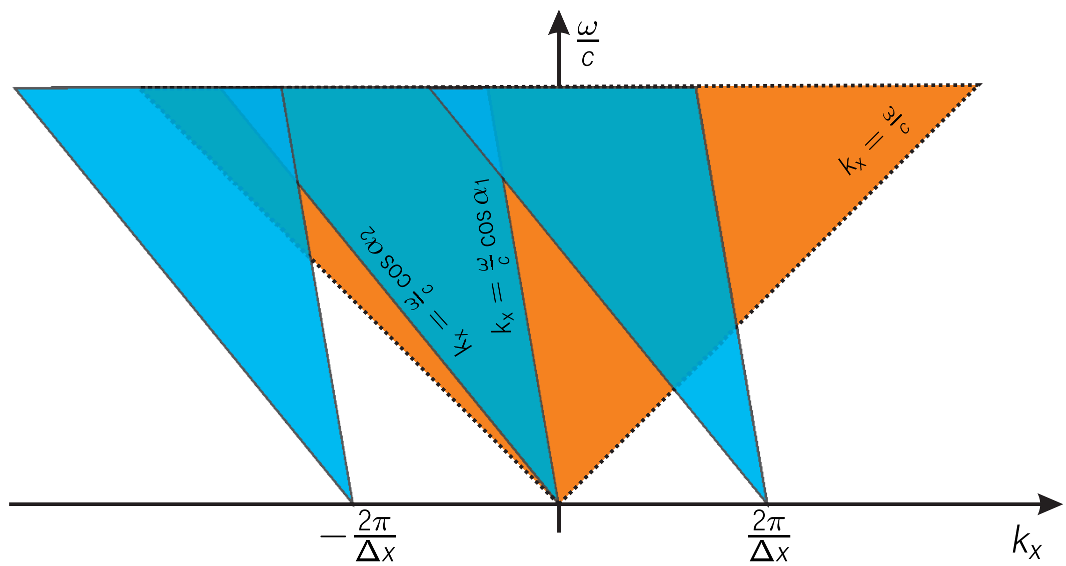
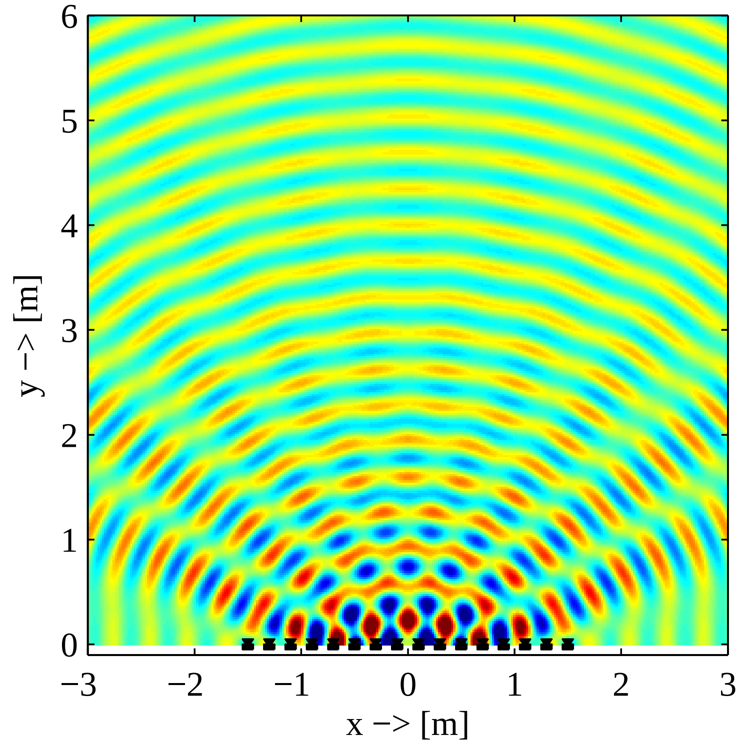


- aliasing frequency $f_{\mathrm{tr,al1}} \geq f_{\mathrm{al}}$ higher compared to infinitely long array
- increased aliasing frequency $f_{\mathrm{tr,al2}} \geq f_{\mathrm{tr,al1}}$ for distant listener positions
- not infinite as for the plane wave case e. g. [Spors et.al, 120th AES]

Example: Truncated Secondary Source Distribution



Aliasing frequencies

$f_{al} \approx 850$ Hz
$f_{tr,al1} \approx 935$ Hz
$f_{tr,al2} \approx 1030$ Hz

$[\mathbf{x}_S = [0 \;\; -1]$ m, $f_s = 1000$ Hz, $\Delta x = 0.20$ m, $N = 16]$

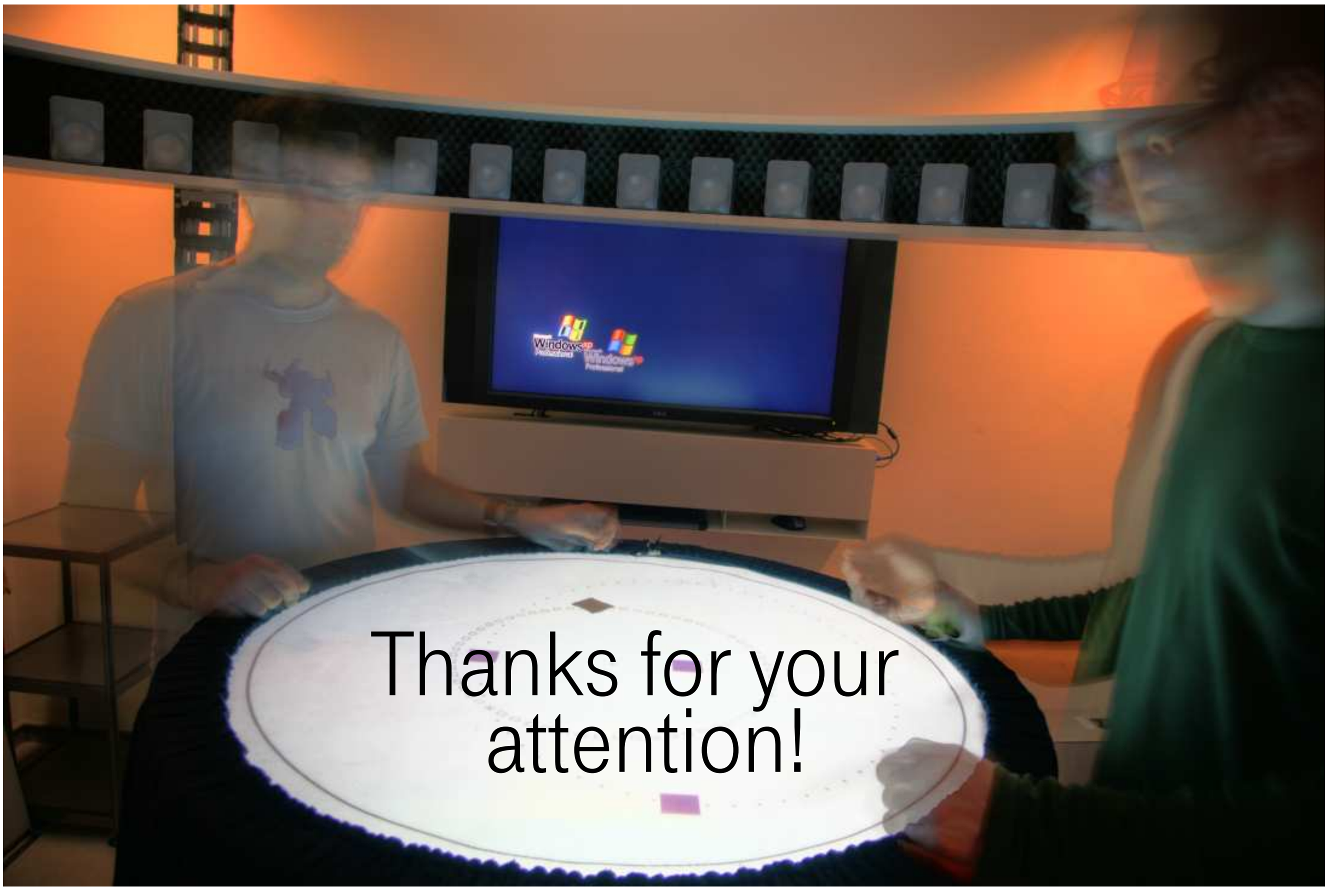
Summary and Conclusions

Main findings

- spatial frequency domain analysis provides interesting insights
- sampling artifacts are listener position dependent in practice
- anti-aliasing conditions for infinte/truncated arrays
- evanescent contributions are side effect of sampling
- application to focused sources [Spors et al., DAGA 2009]

Further work

- artifacts for broadband signals
- extension to 2.5D reproduction (evanescent contributions!)
- perception of evanescent contributions


Thanks for your
attention!