Spatial Sampling Artifacts of Wave Field Synthesis for the Reproduction of Virtual Point Sources

Sascha Spors and Jens Ahrens

Deutsche Telekom Laboratories
Quality and Usability Laboratory
Technische Universität Berlin

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Introduction

Motivation

- Wave field synthesis (WFS) suffers from spatial aliasing in practical implementations
- Spatial aliasing artifacts for plane waves are well understood
- Virtual point sources are frequently used in WFS
- Here: Detailed analysis of spatial aliasing artifacts for virtual point sources
Wave Field Synthesis for Linear Arrays

Rayleigh’s first integral formula provides the pressure in one half space $V$ given the directional pressure gradient on the boundary $\partial V$ of that half space:

$$P(x, \omega) = -2 \int_{-\infty}^{\infty} \frac{\partial}{\partial n} P(x_0, \omega) G_{0,2D}(x - x_0, \omega) \, dx_0$$

The field of a (primary) source $S(x, \omega)$ within the area $V$ is uniquely given by its pressure gradient on the boundary $\partial V$:

$$P(x, \omega) = -2 \int_{-\infty}^{\infty} \frac{\partial}{\partial n} S(x_0, \omega) G_{0,2D}(x - x_0, \omega) \, dx_0$$
Wave Field Synthesis for Linear Arrays

The Green’s function can be interpreted as (secondary) source that generates the field of a desired virtual source \( S(x, \omega) \) inside the listening area \( V \).

\[
P(x, \omega) = \int_{-\infty}^{\infty} D(x_0, \omega) G_{0,2D}(x - x_0, \omega) \, dx_0
\]

\[
D(x_0, \omega) = -2 \frac{\partial}{\partial n} S(x_0, \omega)
\]

Virtual source \( S(x, \omega) \)

\[
p = [x \, y]^T \quad x_0 = [x_0 \, 0]^T
\]

Secondary Source Types

The explicit form of the Green’s function depends on the dimensionality

- two-dimensional reproduction → secondary line sources
- three-dimensional reproduction → secondary point sources

WFS uses secondary point sources for two-dimensional reproduction

- mismatch of secondary source type
- results in amplitude and spectral errors
- artifacts have no influence on sampling theorems (...but on near-field effects)

⇒ analysis of sampling artifacts based on theory of two-dimensional WFS
Spatio-temporal Frequency Domain Description

Reproduced wave field using a linear distribution of secondary sources

\[ P(\mathbf{x}, \omega) = \int_{-\infty}^{\infty} D(x_0, \omega) G_0(\mathbf{x} - \mathbf{x}_0, \omega) \, dx_0 \]

Spatial Fourier transformation \( \mathcal{F}_x \)

\[ \tilde{P}(k_x, y, \omega) = \tilde{D}(k_x, \omega) \cdot \tilde{G}_0(k_x, y, \omega) \]

- spatio-temporal frequency domain description of reproduced wave field
- provides insights into structure of spatial aliasing
- allows to identify the spatial aliasing components

Example: Spectrum of Driving Function / Secondary Sources

- propagating for \( |k_x| \leq \frac{\omega}{c} \), evanescent for \( |k_x| > \frac{\omega}{c} \), \( \omega = 2\pi f \)
Sampling of Secondary Source Distribution

Spatial sampling of continuous secondary source distribution

\[ P(x, \omega) \]

\[ x_0 \]

\[ \Delta x \]

virtual source

\[ S(x, \omega) \]

\[ \Delta x: \text{spatial sampling interval} \]

Sampling of virtual point sources

\[ \tilde{G}_0(k_x, y, \omega) \]

Spatially discrete distribution of secondary sources

- sampling leads to repetitions of spectrum of driving function
- weighted by secondary source spectrum (analog to interpolation)
Spectrum of Reproduced Wave Field

Qualitative illustration of spectra (magnitude) of driving function and secondary sources

- exact reproduction of desired wave field without sampling
- spectra not strictly bandlimited in $k_x$ for given frequency $\omega$
- spectral repetitions due to spatial sampling
- mixtures of propagating/evanescent contributions
Theory of Secondary Source Sampling

Artifacts due to the secondary source sampling can only be avoided when

- the spectrum of the driving function is band-limited, and
- the spectrum of the secondary sources is band-limited.

Consequences of missing band-limitation

- spectral overlaps in the driving function → aliasing
- repetitions weighted by secondary source spectrum → reconstruction error

Anti-aliasing condition (considering the propagating parts only)

\[ f_{\text{al}} \leq \frac{c}{2 \Delta x} \]

Example: \( \Delta x = 0.20 \text{ m} \Rightarrow f_{\text{al}} \approx 850 \text{ Hz} \)

Example: Sampling Artifacts

\[ [x_s = [0 \ -1] \text{ m}, f_s = 2000 \text{ Hz}, \Delta x = 0.20 \text{ m}] \]
Contributions to Reproduced Wave Field

Four contributions to the reproduced spectrum can be identified

<table>
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<tr>
<th>Driving Function</th>
<th>Propagating</th>
<th>Evanescent</th>
</tr>
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<tr>
<td>Secondary Source</td>
<td>Propagating</td>
<td>Propagating 1</td>
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<td></td>
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<td>Evanescent 1</td>
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<tr>
<td></td>
<td></td>
<td>Evanescent 2</td>
</tr>
</tbody>
</table>

- evanescent contributions decay rapidly with distance to secondary sources
- evanescent contributions and propagating 2 occur due to spatial sampling
- perceptual relevance of evanescent contributions unclear
- propagating 1 is main contribution

Example: Contributions to Reproduced Wave Field

\[ [x_s = [0 \ -1] \text{ m}, f_s = 2000 \text{ Hz}, \Delta x = 0.20 \text{ m}] \]
Example: Contributions to Reproduced Wave Field

\[ x_s = [0 \quad -1] \text{ m}, \quad f_s = 2000 \text{ Hz}, \quad \Delta x = 0.20 \text{ m} \]

Truncation of Secondary Source Distribution

- model truncation by spatial windowing of driving function
- convolution with transformed window function in spatial frequency domain
- truncation artifacts (bending waves)

geometric approximation

driving function \( x_{s2} \)
Spectrum of Reproduced Wave Field for Truncated Array

Qualitative illustration of spectra (magnitude) of driving function and secondary sources

- aliasing frequency $f_{\text{tr},a1} \geq f_{a1}$ higher compared to infinitely long array

- increased aliasing frequency $f_{\text{tr},a2} \geq f_{\text{tr},a1}$ for distant listener positions
- not infinite as for the plane wave case e.g. [Spors et al., 120th AES]
Example: Truncated Secondary Source Distribution

\[ x \rightarrow [m] \]
\[ y \rightarrow [m] \]
\[ x_s = [0 \ - \ 1] m, f_s = 1000 \text{ Hz}, \Delta x = 0.20 \text{ m}, N = 16 \]

Aliasing frequencies

\[ f_{al} \approx 850 \text{ Hz} \]
\[ f_{tr,al1} \approx 935 \text{ Hz} \]
\[ f_{tr,al2} \approx 1030 \text{ Hz} \]

Summary and Conclusions

Main findings

- spatial frequency domain analysis provides interesting insights
- sampling artifacts are listener position dependent in practice
- anti-aliasing conditions for infinite/truncated arrays
- evanescent contributions are side effect of sampling
- application to focused sources [Spors et al., DAGA 2009]

Further work

- artifacts for broadband signals
- extension to 2.5D reproduction (evanescent contributions!)
- perception of evanescent contributions
Thanks for your attention!