Reproduction of Focused Sources by the Spectral Division Method

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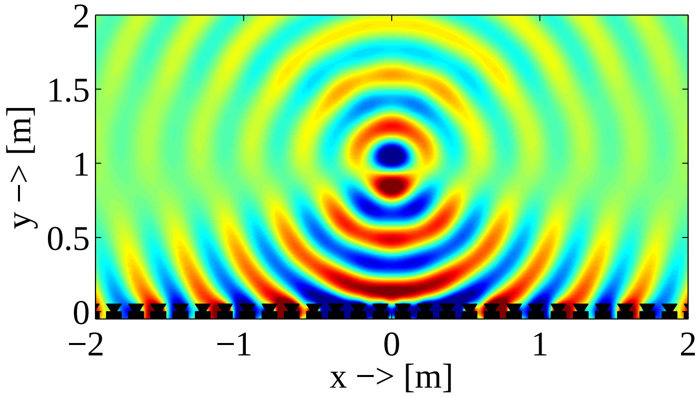
Motivation

- focused sources convey the impression of a virtual source within the listening area
- stunning effect of wave field reconstruction techniques (with limitations)
- state of the art in Wave Field Synthesis (WFS) and Higher-Order Ambisonics (HOA)
- extension of the spectral division method (SDM) towards focused sources

Example: Acoustic Focusing in WFS



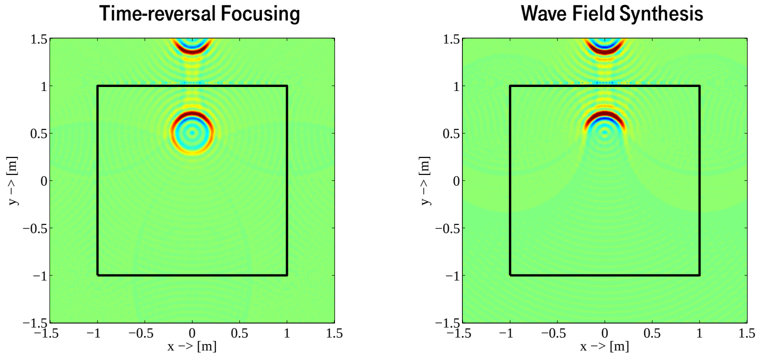
Acoustic Focusing Techniques

Time-reversal Acoustic Focusing

- based on reciprocity of wave equation
- aims at accumulation of energy in time and space
- direction of wave propagation not explicitly taken into account

Focusing Approach in WFS / HOA

- converging wave field towards focus point, diverging after
- source must be located between listeners and loudspeakers for correct auralization
- sensible selection of active secondary sources (listener dependent)



Introduction Foundations of the SDM Focused Sources in the SDM Spatial Sampling Conclusions

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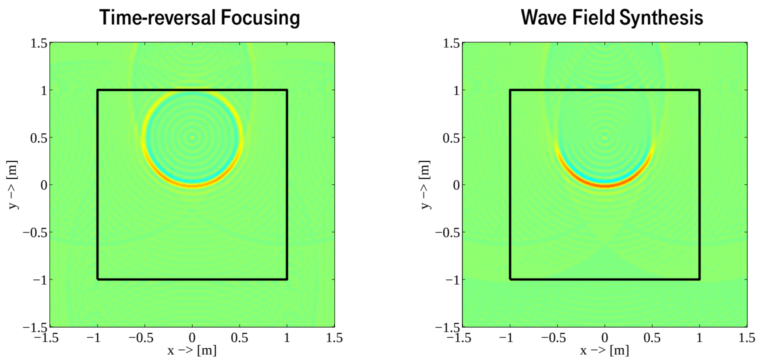

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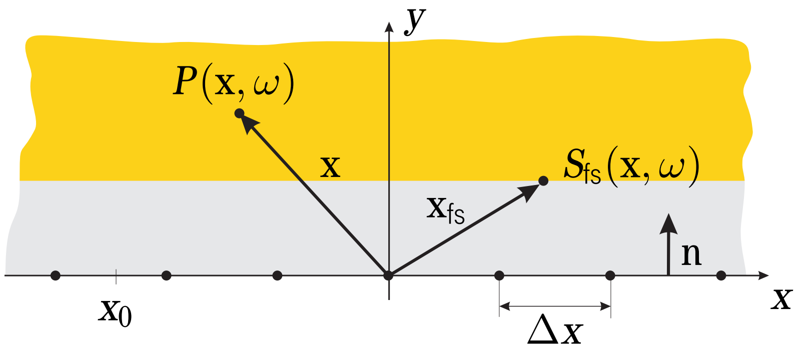
SDM – Basic Concept

Pressure field produced by a linear distribution of weighted secondary sources

$$P(\mathbf{x},\omega) = \int_{-\infty}^{\infty} D(\mathbf{x}_0,\omega)\, G_0(\mathbf{x}-\mathbf{x}_0,\omega)\; d x_0$$



$$\mathbf{x} = [x\ y\ 0]^T$$
$$\mathbf{x}_0 = [x_0\ 0\ 0]^T$$
$$\mathbf{x}_{fs} = [x_{fs}\ y_{fs}\ 0]^T$$

- desired $P(\mathbf{x},\omega) = S_{fs}(\mathbf{x},\omega)$ within listening area
- solution of integral equation is known to be unique
- derivation of driving signal by spectral division and inverse Fourier transformation

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Spatial Fourier transformation $\mathcal{F}_x$

$$\tilde{P}(k_x,y,\omega) = \tilde{D}(k_x,\omega)\ \cdot\ \tilde{G}_0(k_x,y,\omega)$$

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Secondary Source Model

Acoustic point sources are a practical model for secondary sources

$$G(\mathbf{x}-\mathbf{x}_0,\omega) = \frac{1}{4\pi}\frac{e^{-j\frac{\omega}{c}|\mathbf{x}-\mathbf{x}_0|}}{|\mathbf{x}-\mathbf{x}_0|}$$

Spatial Fourier transformation $\mathcal{F}_x$

$$\tilde{G}(k_x,y,\omega) = \begin{cases} -\frac{j}{4}H_0^{(2)}\left(\sqrt{(\frac{\omega}{c})^2-k_x^2}\;y\right) & , \left|\frac{\omega}{c}\right| > |k_x| \\ \frac{1}{2\pi}K_0\left(\sqrt{k_x^2-(\frac{\omega}{c})^2}\;y\right) & , \left|\frac{\omega}{c}\right| < |k_x| \end{cases}$$

- Fourier transformation valid for $y > 0$
- traveling wave for $\left|\frac{\omega}{c}\right| > |k_x|$, evanescent for $\left|\frac{\omega}{c}\right| < |k_x|$

Model of Focused Source

Field of acoustic point source placed within the listening area (for $y > y_{\text{fs}} > 0$)

$$S_{\text{fs}}(\mathbf{x},\omega) = \hat{S}_{\text{fs}}(\omega)\frac{1}{4\pi}\frac{e^{-j\frac{\omega}{c}|\mathbf{x}-\mathbf{x}_{\text{fs}}|}}{|\mathbf{x}-\mathbf{x}_{\text{fs}}|}$$

Spatial Fourier transformation $\mathcal{F}_x$

$$\tilde{S}_{\text{fs}}(k_x,y,\omega) = \hat{S}_{\text{fs}}(\omega)\;e^{jk_x x_{\text{fs}}}\begin{cases} -\frac{j}{4}H_0^{(2)}\left(\sqrt{(\frac{\omega}{c})^2-k_x^2}\;(y-y_{\text{fs}})\right) & , \left|\frac{\omega}{c}\right| > |k_x| \\ \frac{1}{2\pi}K_0\left(\sqrt{k_x^2-(\frac{\omega}{c})^2}\;(y-y_{\text{fs}})\right) & , \left|\frac{\omega}{c}\right| < |k_x| \end{cases}$$

- no explicit model for $y < y_{\text{fs}}$
- model for $y > y_{\text{fs}}$ suitable due to uniqueness of solution
- alternative: model of point sink for $y < y_{\text{fs}}$

Driving Signal for Focused Sources

The driving signal is yielded by spectral division

$$\tilde{D}_{\text{fs}}(k_x, \omega) = \hat{S}_{\text{fs}}(\omega)\, e^{jk_x x_{\text{fs}}} \begin{cases} \dfrac{H_0^{(2)}\left(\sqrt{(\frac{\omega}{c})^2 - k_x^2}\,(y - y_{\text{fs}})\right)}{H_0^{(2)}\left(\sqrt{(\frac{\omega}{c})^2 - k_x^2}\, y\right)} & , \ \left|\frac{\omega}{c}\right| > |k_x| \\[2ex] \dfrac{K_0\left(\sqrt{k_x^2 - (\frac{\omega}{c})^2}\,(y - y_{\text{fs}})\right)}{K_0\left(\sqrt{k_x^2 - (\frac{\omega}{c})^2}\, y\right)} & , \ \left|\frac{\omega}{c}\right| < |k_x| \end{cases}$$

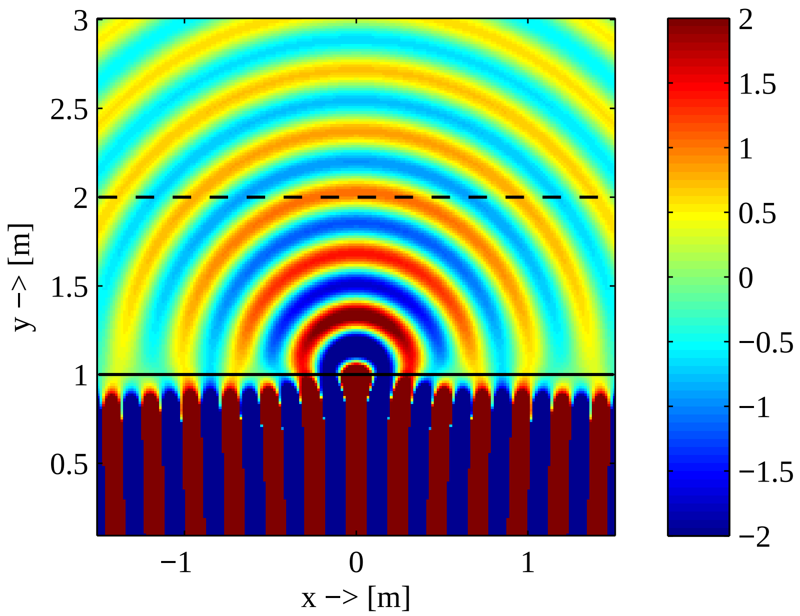
- driving signal depends on listener distance y to secondary source distribution
- inverse Fourier transformation of driving signal not available

2.5-dimensional Reproduction

- evaluate driving signal for a reference distance $y = y_{\text{ref}}$ (reference line)
- correct reproduction only on reference line
- amplitude and (slight) spectral deviations besides reference line

Example – Synthesized Wave Field of a Focused Source

Monochromatic Driving Signal with Evanescent Contributions



($\mathbf{x}_{\text{fs}} = [0\ 1\ 0]^T$ m, $f_{\text{fs}} = 1$ kHz, $y_{\text{ref}} = 2$ m)

Modified Driving Signal

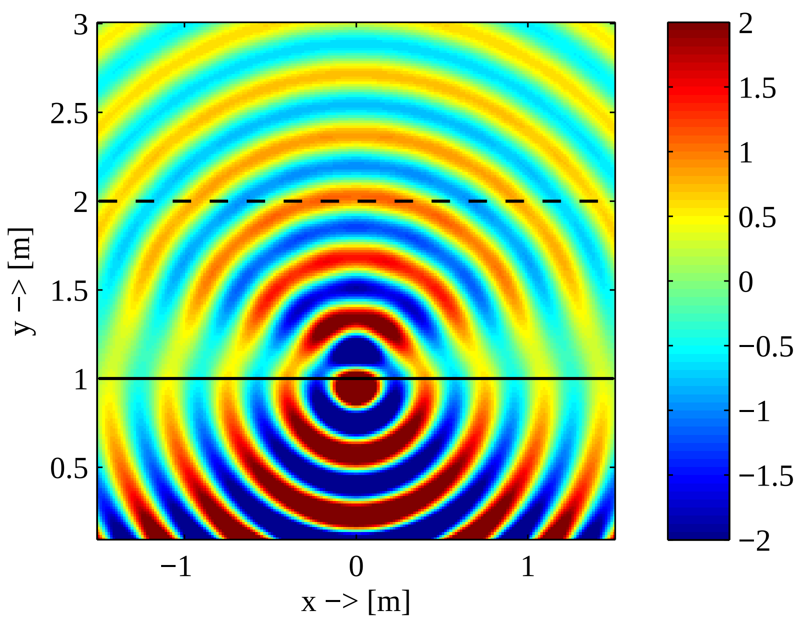
- strong evanescent contributions before focus point ($y < y_{\text{fs}}$)
- consequence of desired evanescent contributions behind focus point ($y > y_{\text{fs}}$)
- idea: discard evanescent contributions in model of focused source

Modified Driving Signal

$$\tilde{D}_{\text{mod,fs}}(k_x,\omega) = \hat{S}_{\text{fs}}(\omega)\, e^{jk_x x_{\text{fs}}} \begin{cases} \frac{H_0^{(2)}\left(\sqrt{(\frac{\omega}{c})^2-k_x^2}\,(y_{\text{ref}}-y_{\text{fs}})\right)}{H_0^{(2)}\left(\sqrt{(\frac{\omega}{c})^2-k_x^2}\,y_{\text{ref}}\right)} & , \left|\frac{\omega}{c}\right| > |k_x| \\ 0 & , \left|\frac{\omega}{c}\right| < |k_x| \end{cases}$$

Example – Synthesized Wave Field of a Focused Source

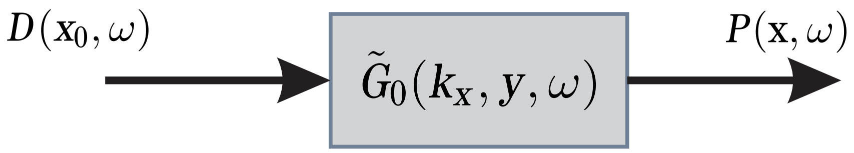
Monochromatic Modified Driving Signal without Evanescent Contributions

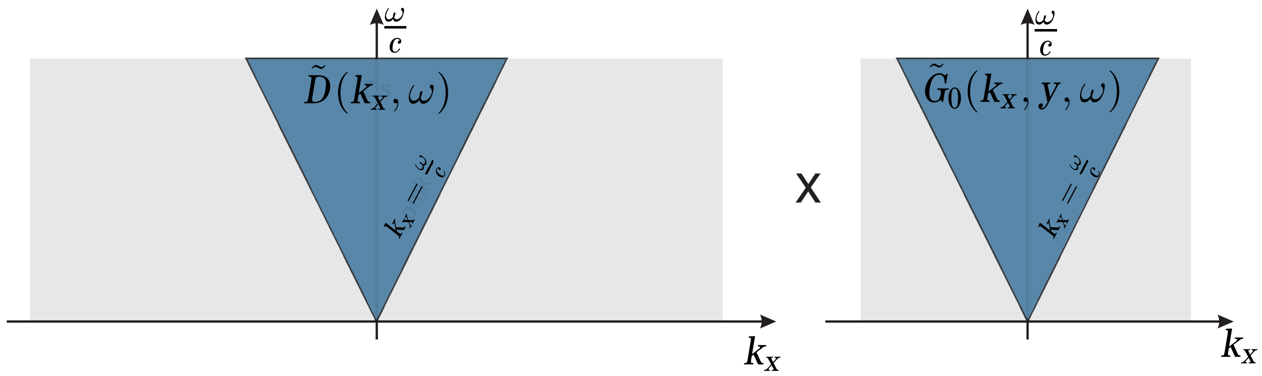


($\mathbf{x}_{\text{fs}} = [0\ 1\ 0]^T$ m, $f_{\text{fs}} = 1$ kHz, $y_{\text{ref}} = 2$ m)

Sampling of Secondary Source Distribution

Continuous distribution of secondary sources





- sampling leads to repetition of spectrum of driving function ⟶ overlaps ⟶ aliasing
- weighted by secondary source spectrum ⟶ reconstruction error

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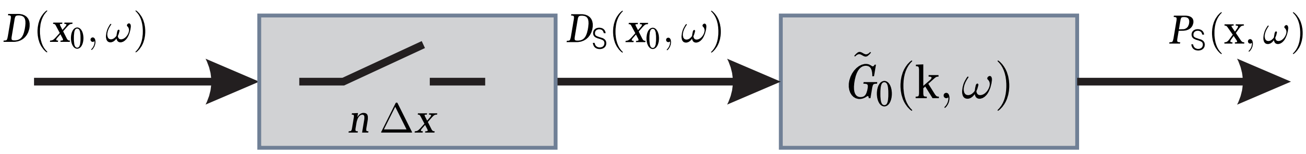
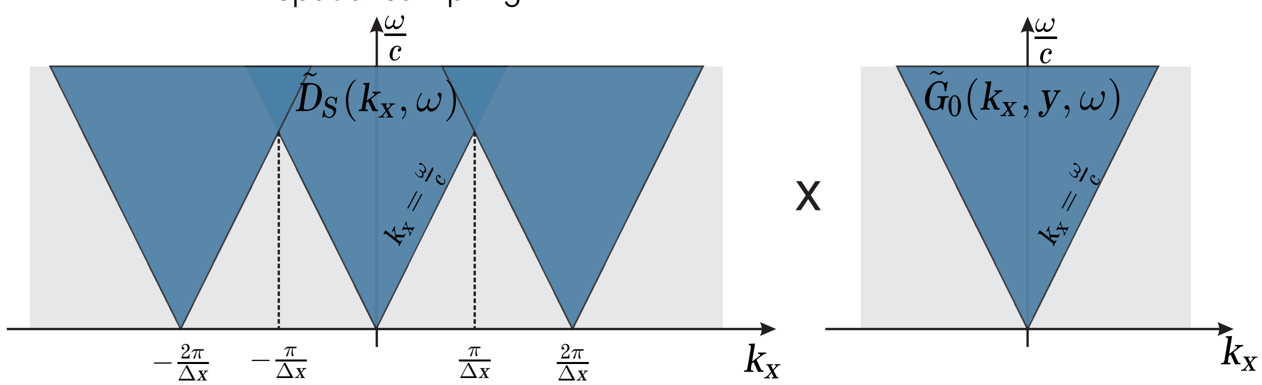
Sampling of Secondary Source Distribution

Spatially discrete distribution of secondary sources



spatial sampling



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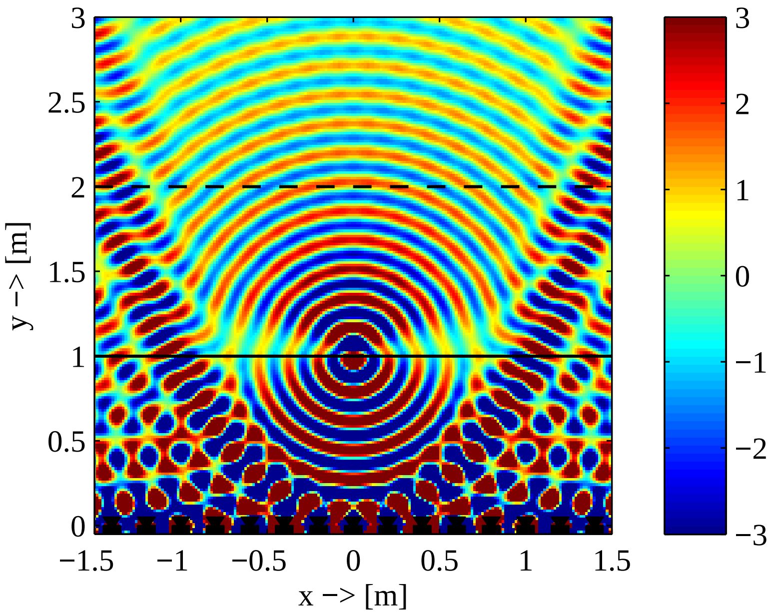
Example – Synthesized Wave Field of a Focused Source

Spatially Discrete Secondary Source Distribution



($\mathbf{x}_{\text{fs}} = [0\ 1\ 0]^T$ m, $f_{\text{fs}} = 1$ kHz, $y_{\text{ref}} = 2$ m, $\Delta x = 0.20$ m)

Summary and Conclusions

Main findings

- SDM provides exact solution to acoustic focusing problem
- not feasible to reproduce evanescent contributions of focused source
- WFS can be interpreted as an approximation of the SDM
  - focused sources show interesting aliasing properties
  - amplitude deviations due to 2.5-dimensional reproduction

Further work

- efficient implementation of driving function
- research on audibility of evanescent contributions
- listening experiment to compare SDM with established methods