Reproduction of Focused Sources by the Spectral Division Method

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4th International Symposium on Communications, Control and Signal Processing
Limassol 2010

Motivation

- focused sources convey the impression of a virtual source within the listening area
- stunning effect of wave field reconstruction techniques (with limitations)
- state of the art in Wave Field Synthesis (WFS) and Higher-Order Ambisonics (HOA)
- extension of the spectral division method (SDM) towards focused sources

Example: Acoustic Focusing in WFS
Acoustic Focusing Techniques

Time-reversal Acoustic Focusing
- based on reciprocity of wave equation
- aims at accumulation of energy in time and space
- direction of wave propagation not explicitly taken into account

Focusing Approach in WFS / HOA
- converging wave field towards focus point, diverging after
- source must be located between listeners and loudspeakers for correct auralization
- sensible selection of active secondary sources (listener dependent)
**SDM – Basic Concept**

Pressure field produced by a linear distribution of weighted secondary sources

\[
P(x, \omega) = \int_{-\infty}^{\infty} D(x_0, \omega) G_0(x - x_0, \omega) \, dx_0
\]

- desired \( P(x, \omega) = S_{fs}(x, \omega) \) within listening area
- solution of integral equation is known to be unique
- derivation of driving signal by spectral division and inverse Fourier transformation
Secondary Source Model

Acoustic point sources are a practical model for secondary sources

\[ G(x - x_0, \omega) = \frac{1}{4\pi} \frac{e^{-j\frac{\omega}{c}|x-x_0|}}{|x-x_0|} \]

Spatial Fourier transformation \( \mathcal{F}_x \)

\[ \tilde{G}(k_x, y, \omega) = \begin{cases} \frac{i}{4}H_0^{(2)}\left(\sqrt{\left(\frac{\omega}{c}\right)^2 - k_x^2} y\right), & \left|\frac{\omega}{c}\right| > |k_x| \\ \frac{1}{2\pi}K_0\left(\sqrt{k_x^2 - \left(\frac{\omega}{c}\right)^2} y\right), & \left|\frac{\omega}{c}\right| < |k_x| \end{cases} \]

- Fourier transformation valid for \( y > 0 \)
- traveling wave for \( \left|\frac{\omega}{c}\right| > |k_x| \), evanescent for \( \left|\frac{\omega}{c}\right| < |k_x| \)

Model of Focused Source

Field of acoustic point source placed within the listening area (for \( y > y_{fs} > 0 \))

\[ S_{fs}(x, \omega) = \hat{S}_{fs}(\omega) \frac{1}{4\pi} \frac{e^{-j\frac{\omega}{c}|x-x_{fs}|}}{|x-x_{fs}|} \]

Spatial Fourier transformation \( \mathcal{F}_x \)

\[ \hat{S}_{fs}(k_x, y, \omega) = \hat{S}_{fs}(\omega) e^{j k_x x_{fs}} \begin{cases} \frac{i}{4}H_0^{(2)}\left(\sqrt{\left(\frac{\omega}{c}\right)^2 - k_x^2} (y - y_{fs})\right), & \left|\frac{\omega}{c}\right| > |k_x| \\ \frac{1}{2\pi}K_0\left(\sqrt{k_x^2 - \left(\frac{\omega}{c}\right)^2} (y - y_{fs})\right), & \left|\frac{\omega}{c}\right| < |k_x| \end{cases} \]

- no explicit model for \( y < y_{fs} \)
- model for \( y > y_{fs} \) suitable due to uniqueness of solution
- alternative: model of point sink for \( y < y_{fs} \)
Driving Signal for Focused Sources

The driving signal is yielded by spectral division

\[
\tilde{D}_{ls}(k_x, \omega) = \tilde{S}_{ls}(\omega) e^{i k_x x_{fs}} \begin{cases} 
\frac{H_0^{(2)}(\sqrt{\left(\frac{\omega}{c}\right)^2 - k_x^2 (y_{fs} - y_f)} - k_x y_f)}{H_0^{(2)}(\sqrt{\left(\frac{\omega}{c}\right)^2 - k_x^2 y_f})} & , \left| \frac{\omega}{c} \right| > |k_x| \\
\frac{K_0(\sqrt{k_x^2 - \left(\frac{\omega}{c}\right)^2 (y_{fs} - y_f)})}{K_0(\sqrt{k_x^2 - \left(\frac{\omega}{c}\right)^2 y_f})} & , \left| \frac{\omega}{c} \right| < |k_x| 
\end{cases}
\]

- driving signal depends on listener distance \( y \) to secondary source distribution
- inverse Fourier transformation of driving signal not available

2.5-dimensional Reproduction

- evaluate driving signal for a reference distance \( y = y_{ref} \) (reference line)
- correct reproduction only on reference line
- amplitude and (slight) spectral deviations besides reference line

Example – Synthesized Wave Field of a Focused Source

Monochromatic Driving Signal with Evanescent Contributions

\( (x_{fs} = [0 1 0]^T \text{ m}, f_{fs} = 1 \text{ kHz}, y_{ref} = 2 \text{ m}) \)
Modified Driving Signal

- strong evanescent contributions before focus point $y < y_{fs}$
- consequence of desired evanescent contributions behind focus point $y > y_{fs}$
- idea: discard evanescent contributions in model of focused source

$$\tilde{D}_{mod,fs}(k_x, \omega) = \tilde{S}_{fs}(\omega) e^{jk_x x_{fs}} \begin{cases} 
H_0^{(2)}(\sqrt{(\frac{\omega}{c})^2 - k_x^2} (y_{ref} - y_{fs})) & |\frac{\omega}{c}| > |k_x| \\
H_0^{(2)}(\sqrt{(\frac{\omega}{c})^2 - k_x^2} y_{ref}) & |\frac{\omega}{c}| < |k_x| 
\end{cases}$$

Example – Synthesized Wave Field of a Focused Source

Monochromatic Modified Driving Signal without Evanescent Contributions

$$(x_{fs} = [0 \ 1 \ 0]^T \text{ m}, f_{fs} = 1 \text{ kHz}, y_{ref} = 2 \text{ m})$$
Sampling of Secondary Source Distribution

Continuous distribution of secondary sources

$$D(x_0, \omega) \quad \tilde{G}_0(k_x, y, \omega) \quad P(x, \omega)$$

Sampling leads to repetition of spectrum of driving function → overlaps → aliasing
- weighted by secondary source spectrum → reconstruction error

Spatially discrete distribution of secondary sources

$$D(x_0, \omega) \quad D_S(x_0, \omega) \quad n \Delta x \quad \tilde{G}_0(k, \omega) \quad P_S(x, \omega)$$

Spatial sampling

Sampling leads to repetition of spectrum of driving function → overlaps → aliasing
- weighted by secondary source spectrum → reconstruction error
Example – Synthesized Wave Field of a Focused Source
Spatially Discrete Secondary Source Distribution

\[(x_{fs} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T \text{m}, f_{fs} = 1 \text{kHz}, y_{ref} = 2 \text{ m}, \Delta x = 0.20 \text{ m})\]

Summary and Conclusions

Main findings
- SDM provides exact solution to acoustic focusing problem
- not feasible to reproduce evanescent contributions of focused source
- WFS can be interpreted as an approximation of the SDM
  - focused sources show interesting aliasing properties
  - amplitude deviations due to 2.5-dimensional reproduction

Further work
- efficient implementation of driving function
- research on audibility of evanescent contributions
- listening experiment to compare SDM with established methods