DIRECTIONLETS: ANISOTROPIC MULTI-DIRECTIONAL REPRESENTATION WITH SEPARABLE FILTERING

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ABSTRACT

In spite of the success of the standard wavelet transform (WT) in image processing, the efficiency of its representation is limited by the spatial isotropy and only the horizontal and vertical directions used in the construction. We present a new *anisotropic multi-directional perfect reconstruction* and *critically sampled* transform that retains the simplicity of the standard WT and improves the efficiency of image representation.

1. INTRODUCTION

The problem of finding efficient representation of images is a fundamental problem in many image processing areas, like approximation and compression. An efficient transform-based representation requires sparsity, that is a large amount of information has to be contained in a small portion of transform coefficients.

The 1-D WT has become very successful in the last decade because it provides an efficient multiresolution representation of 1-D piecewise smooth signals [1]. The application of wavelets to image processing requires the design of 2-D WT and the most common approach is to use 2-D separable filter-banks, which consists of the direct product of two independent 1-D filter-banks in the horizontal and vertical directions. This method is conceptually simple and has low complexity.

However, the standard 2-D WT fails to provide a compact representation of 1-D discontinuities, like edges and contours. To capture efficiently these geometrical features, the basis functions are required (a) to be *anisotropic* and (b) to have *directional vanishing moments* (DVM). Several approaches have been proposed so far (like curvelets, contourlets and bandelets), but they often require *oversampling*, have *higher complexity* than the standard 2-D WT, and require *non-separable* convolution and filter design.

Our goal is to construct anisotropic perfect reconstruction and critically sampled basis functions with DVM along different directions, while retaining the simplicity of separable processing and filter design from the standard 2-D WT. We call these basis functions *directionlets*. We show that our transform has good approximation and compression properties as compared to other overcomplete transforms and is superior to the performance of the standard 2-D WT, while having the same complexity.

2. CONSTRUCTION OF DIRECTIONLETS

Directionlets are constructed as basis functions of the so-called *skewed anisotropic wavelet transforms* (S-AWT). These transforms provide anisotropy and impose DVM in the basis functions along *any* two directions with rational slopes.

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(a) (b) (c)

Fig. 1. (a) Comparison between the standard and our zerotree-based compression methods at low bit rates. Reconstructions are obtained at the same bit rate (0.04bpp) using (b) the standard 2-D WT (23.5dB) and (c) directionlets (26.1db).

The construction of the S-AWT is based on integer lattices. The S-AWT consists of applying the 1-D WT steps along the corresponding directions of the lattice, where the number of steps along the two directions is not necessarily equal.[1]

3. APPROXIMATION AND COMPRESSION OF IMAGES

Directionlets have been shown [2] to improve substantially the efficiency of representation of images that contain anisotropic structures in different orientations. In the case of non-linear approximation of synthetic piecewise smooth images with one smooth 1-D discontinuity, directionlets can achieve the rate of decay of mean-square error of the order $O(N^{-1.55})$, for N retained coefficients [2]. This order outperforms the corresponding order achieved by the standard 2-D WT. It is also comparable to the orders achieved by other similar *overcomplete* transforms, while having lower complexity.

Since directionlets are critically sampled, the implementation in the wavelet-based compression methods is straightforward. Here, we present compression results obtained by the zerotree coding algorithm using directionlets instead of the standard 2-D WT and allowing for spatial segmentation and adaptation of the transform directions to the content of each segment. The optimal segmentation and the choice of transform directions are found using the Lagrangian optimization. Our compression method outperforms significantly the original zerotree algorithm, especially at low bit rates, as shown in Fig. 1.

4. REFERENCES

[1] M. Vetterli and J. Kovačević, *Wavelets and Subband Coding*. New Jersey, NJ: Prentice Hall PTR, 1995.

[2] V. Velisavljević, B. Beferull-Lozano, M. Vetterli, and P. L. Dragotti, "Directionlets: Anisotropic multi-directional representation with separable filtering," *IEEE Trans. Image Processing*, to appear.

[1]Here, for the reason of the lack of space, we give only a very brief explanation of the construction (see [2] for more details).