How to Exploit Directional Features in Images

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Outline

- **Motivation for the multi-directional wavelet transform**
- Construction of the M-Dir WT with 1-D filters
- Basis functions and regularity
- Applications:
  - Non-linear approximation and compression of images
  - Denoising of images

Motivation

Directional information in images are distributed not only across the horizontal and vertical directions.

A. Milkowski, *Diamond #1*

S. Paz, *Dimensional Love #8*

Karen's gallery, *Multi-directional scarf*

The standard 2-D wavelet transform is widely used.

- **Advantages:** sparse representation, separability, simple filter design, low computational complexity.
- **Disadvantages:** only the horizontal & vertical directional information is captured.

Motivation

Characterization of features in images requires analysis in different directions (not only horizontal & vertical).

Several multi-directional approaches have already been proposed:

- *Curvelets* (Candès and Donoho),
- *Contourlets* (Do et al.),
- *Wedgelets* (Donoho, Baraniuk),
- *Wedgeprints* (Wakin et al.),
- *Edgeprints* (Dragotti & Vetterli),
- *Bandelets* (Mallat),
- *Polynomial modeling & quadtree segmentation* (Shukla et al.),
- *Steerable pyramid* (Simoncelli),
- *Directional filter banks* (Bamberger & Smith),
- *Complex wavelets* (Kingsbury),
- *Directional wavelets* (Zuidwijk).

Disadvantages: Non-separability, continuous-time filter design, high computational complexity

Our goals: Simplicity, separable filter design, computational efficiency

Idea: Get multi-directionality while keeping 1-D filtering only

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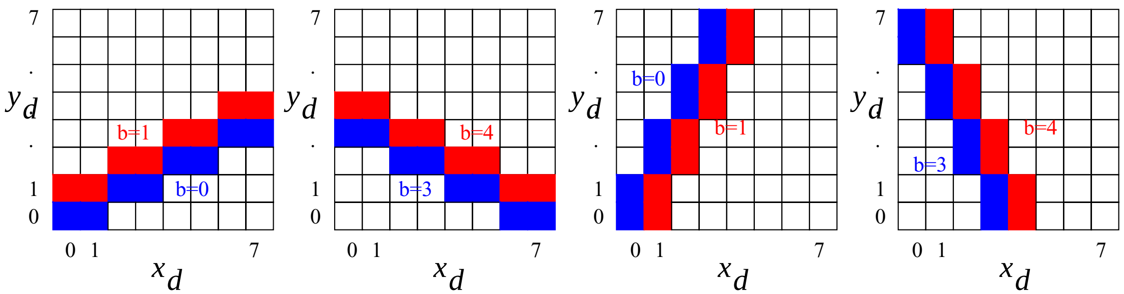
Digital Lines

Approximate continuous line

$$y_c(x_y) = rx_c + b$$

by digital line (Bresenham, 1965)

$$y_d = \lfloor rx_d \rfloor + \lfloor b \rfloor.$$

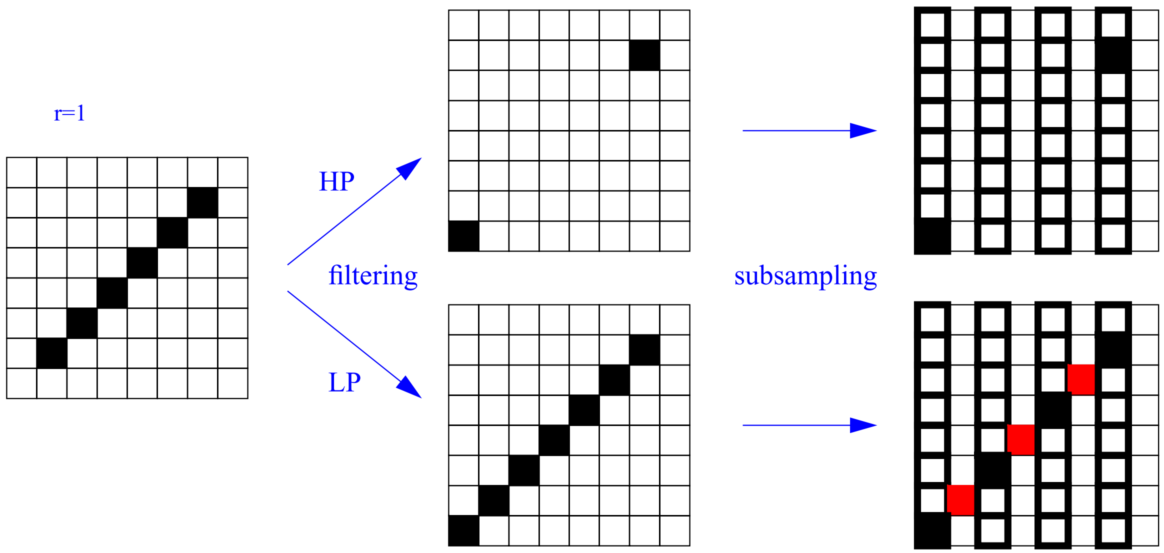


$r = \frac{1}{2}$ $r = -\frac{1}{2}$ $r = 2$ $r = -2$

Complete partition of the discrete space $\mathbb{Z}^2$ is ensured.

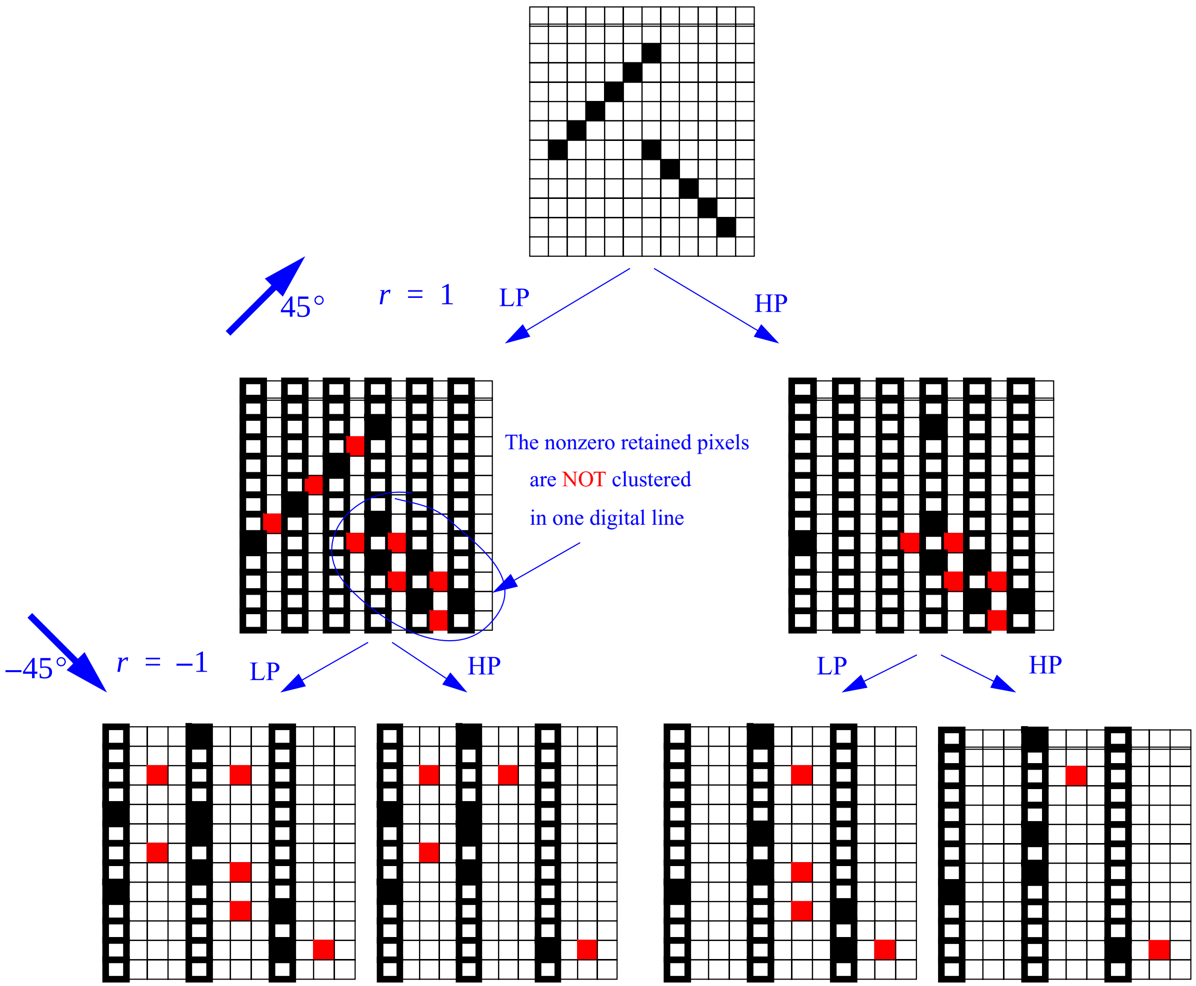
Digital Lines

- 1-D wavelet filter bank applied along parallel lines
- Directional zero moments imposed in the high-pass subband



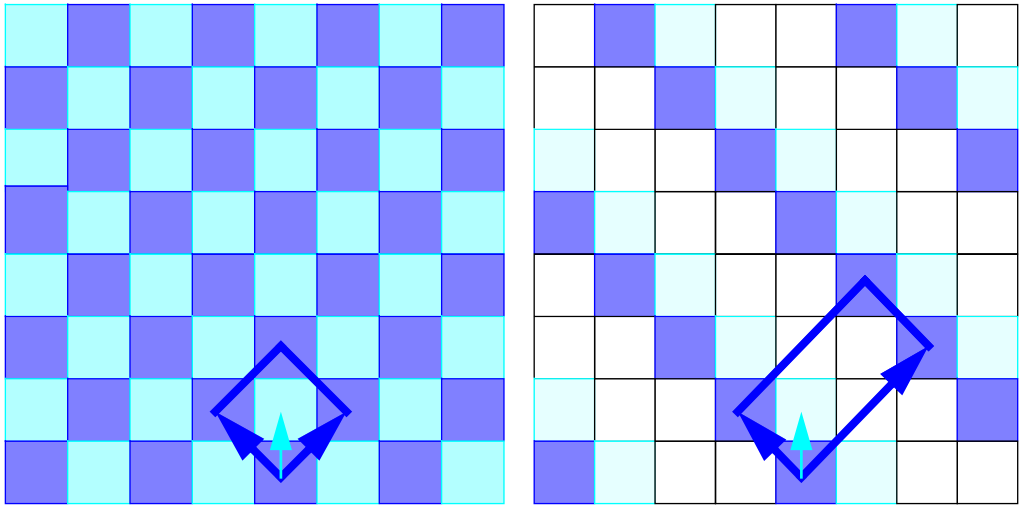
Problem: Filtering based on digital lines produces directional interaction.

Directional Interaction



Lattice-Based Filtering

- Filtering applied along pixels belonging to lattice Λ defined by M_Λ
- Processing is performed independently in each coset

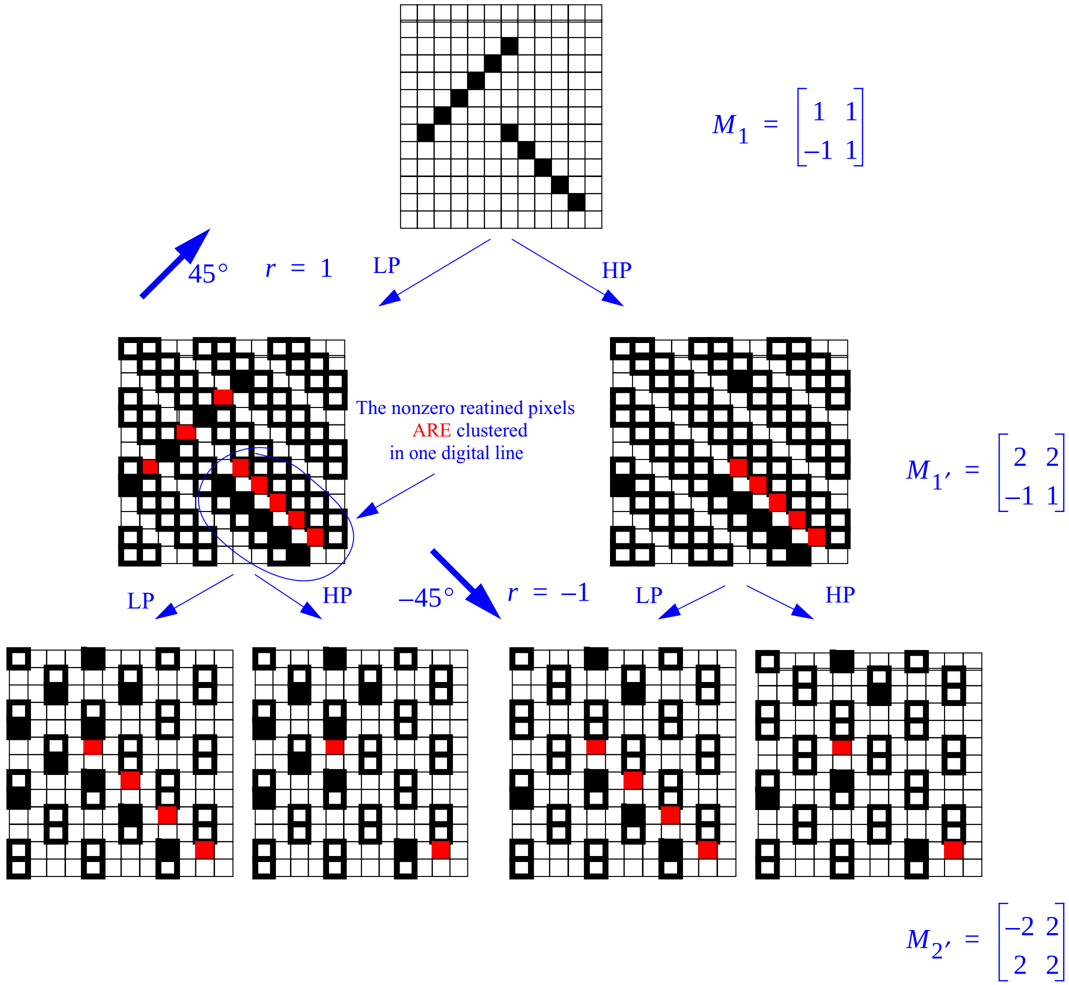


$$M_\Lambda = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \qquad M_{\Lambda'} = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$$

$$c_0 = \begin{bmatrix} 0 & 0 \end{bmatrix} \quad c_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

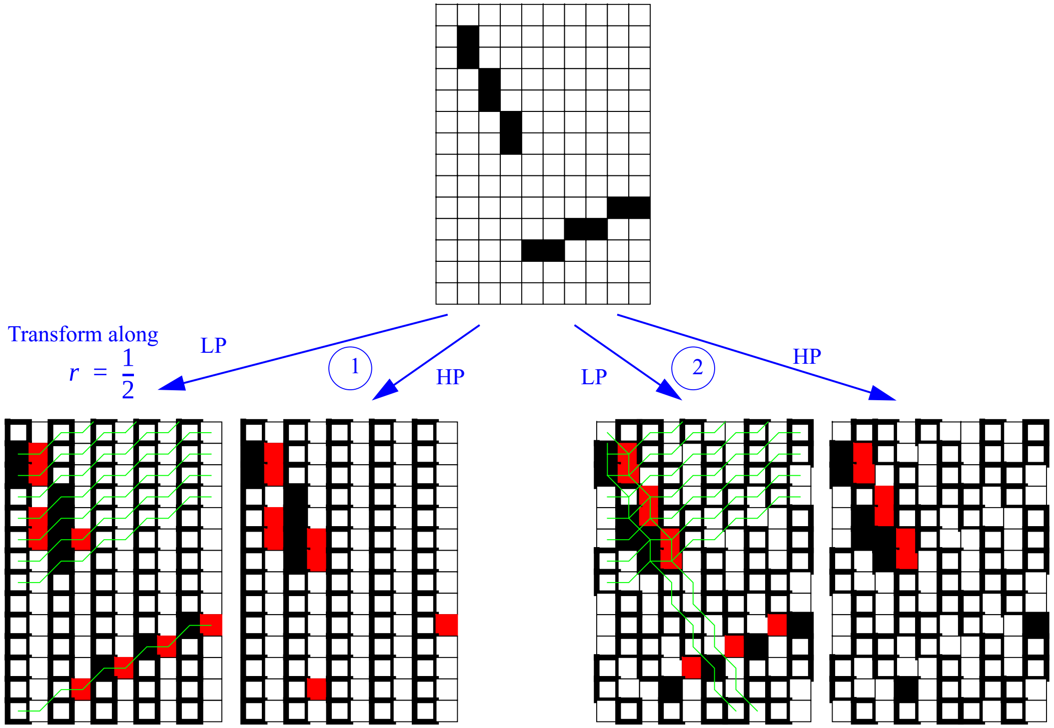
Lattice-based filtering does not suffer from directional interaction.

Lattice-Based Filtering



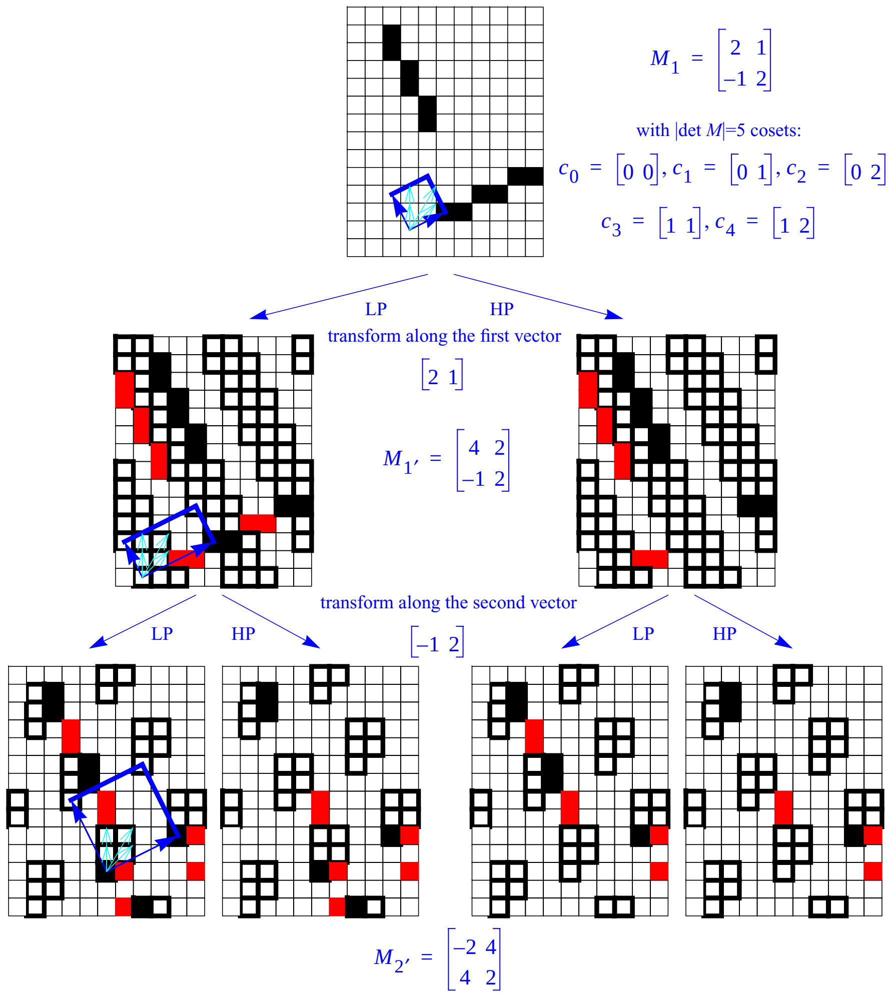
Alternative vs. Lattice-Based Filtering

The only method preventing directional interaction (for any transform direction) is lattice-based filtering



Not possible to keep pixels aligned

Alternative vs. Lattice-Based Filtering

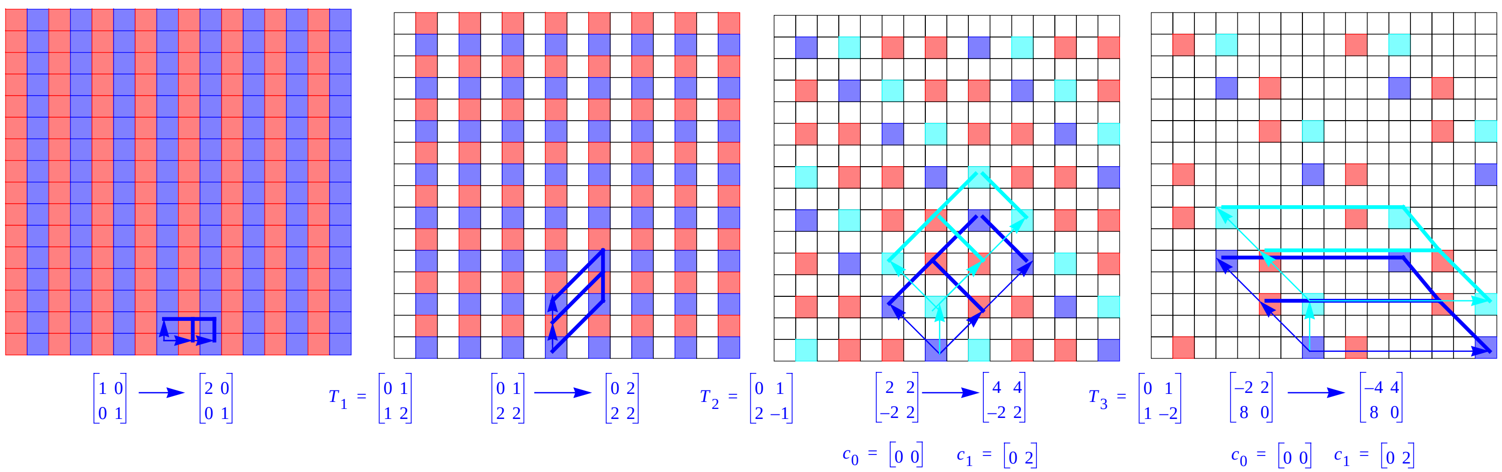


Lattice-Based Construction: Iterated Filtering

1) Filtering along a chosen transform direction: $\Lambda_i, M_i = \begin{pmatrix} d_i^{(1)} \\ d_i^{(2)} \end{pmatrix}$.

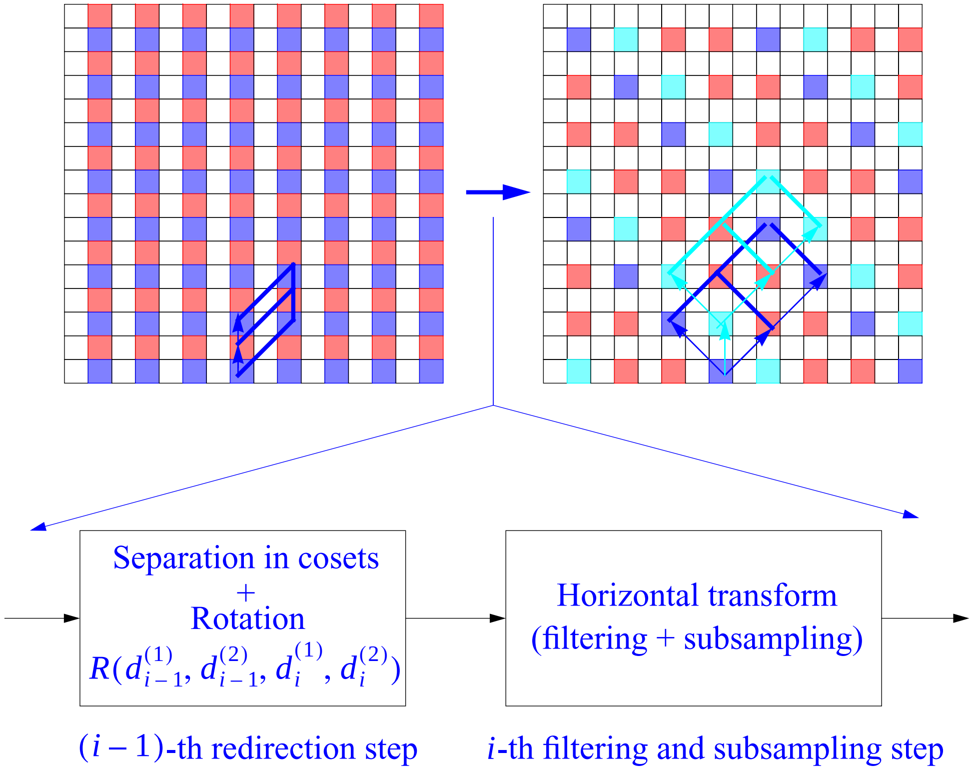
2) Subsampling along the same direction: $\Lambda_i', M_i' = \begin{pmatrix} 2d_i^{(1)} \\ d_i^{(2)} \end{pmatrix}$.

3) Redirection: $\Lambda_{i+1}, M_{i+1} = \begin{pmatrix} d_{i+1}^{(1)} \\ d_{i+1}^{(2)} \end{pmatrix} = T_i \cdot M_i'$.



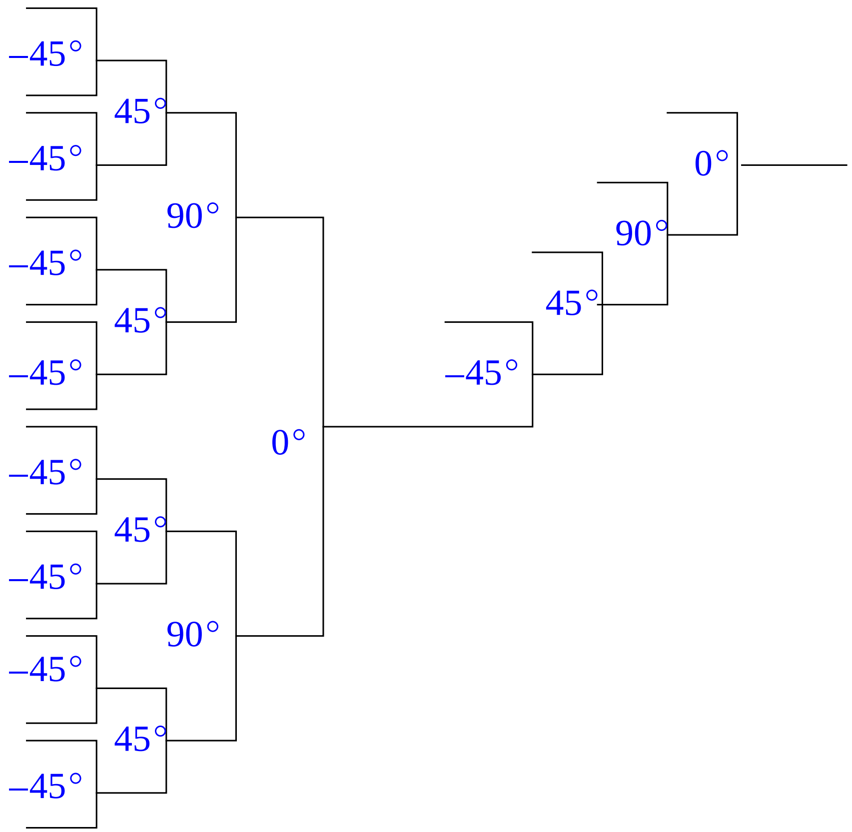
Lattice-Based Construction: Rotation

Another way of looking at it:



Cyclic Construction

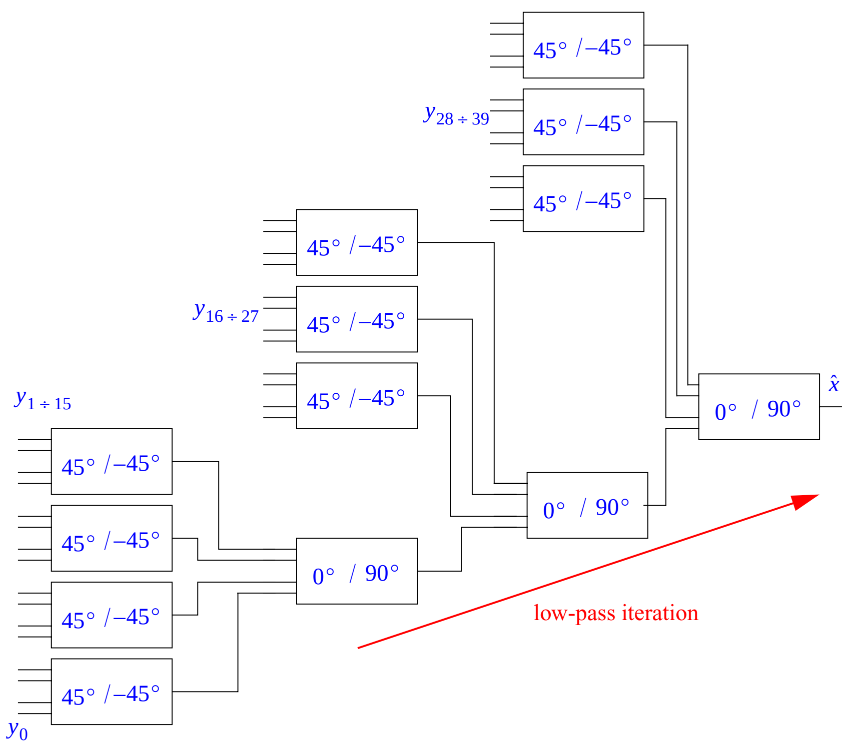
The order of transform directions determines the MDir WT.



Transform directions are repeated periodically

Pairs of Transform Directions

Arbitrary choice of transform directions in cyclic construction may lead to non-smooth basis functions.



However, iteration of any two transform directions ensures smoothness.

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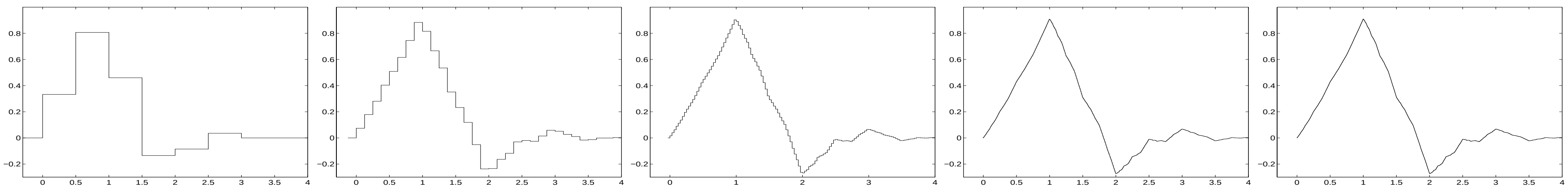
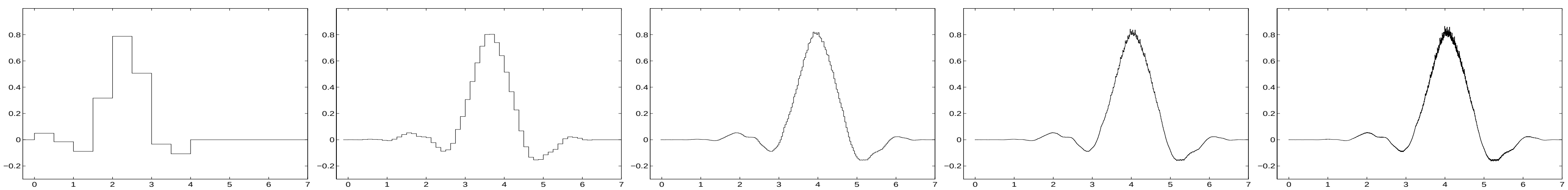
Regularity

Regularity (*smoothness*) imposes convergence of iterated discrete filters to continuous function

- Regular filter (*Daubechies D-4*)



- Irregular filter (*Smith-Barnwell*)



Motivation for Regularity

Why is regularity important in image processing applications?

1) No artificial discontinuities appear in transform domain.

2) Perturbation of transform coefficients $\longrightarrow$ smooth perturbation in image domain.

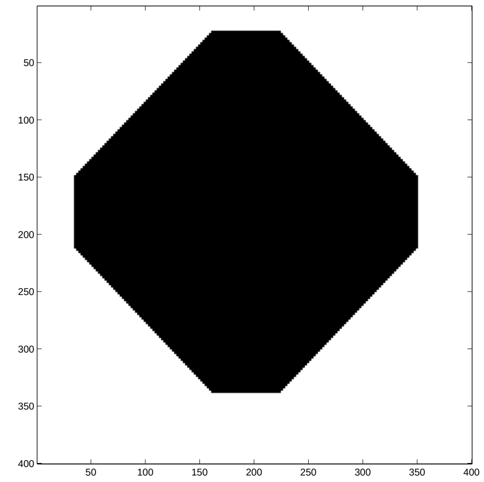
Conditions:

- time-domain: $\max_n |g_0^{(i)}[n+1] - g_0^{(i)}[n]| \leq C2^{-i\alpha}$
- frequency-domain: $G_0(e^{j\omega}) = O\left(\frac{1}{(1+|\omega|)^\alpha}\right)$

(Rioul, Cohen, Daubechies)

Regularity in Cyclic Construction

- The cyclic construction is symmetric
- Support of basis functions is also symmetric



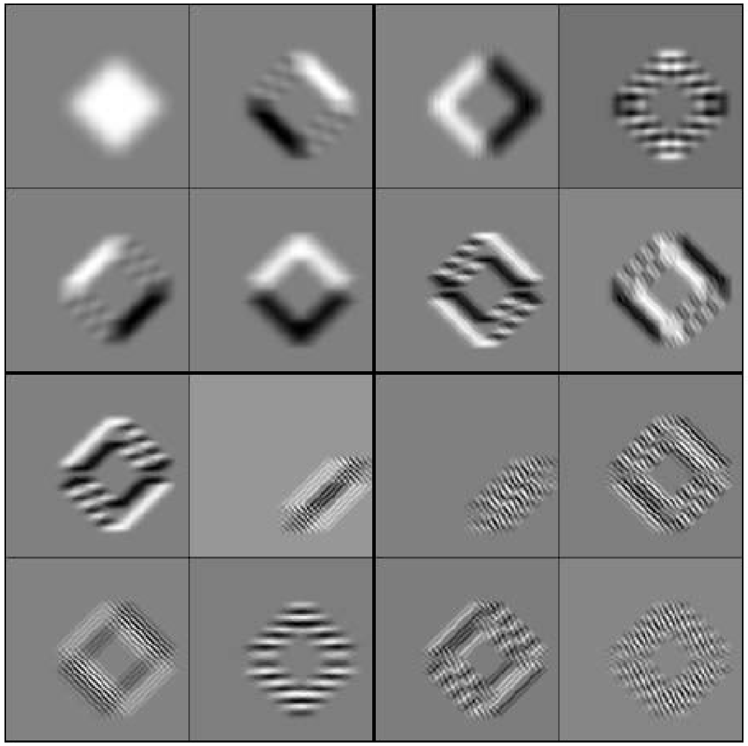
One way to ensure regularity:

- take a regular 1-D lowpass filter $G_0'(z)$ and build an extended filter $G_0(z)$ as

$$G_0(z) = \prod_{i=0}^{m-1} G_0'\left(z^{2^i}\right), \qquad m = N/2 + \frac{1}{2}\sum_{j=1}^{N} \log_2 |\det T_i|,$$

where N is the period of the cycle.

Regularity vs. Orthogonality

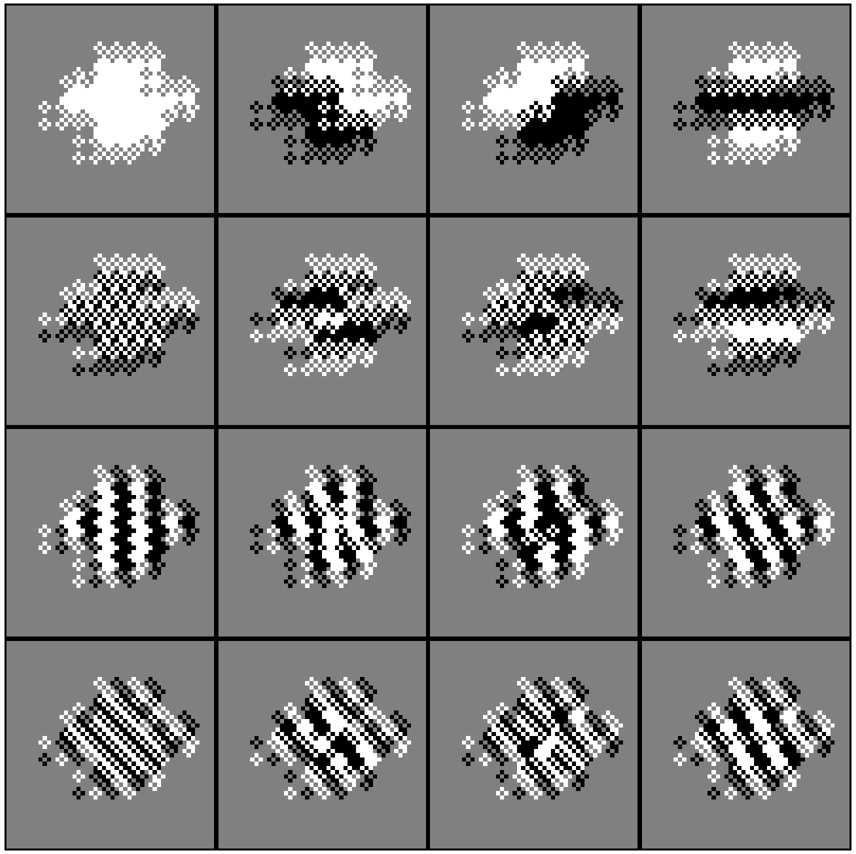


$$G_0(z) = \prod_{i=0}^{m-1} G'_0\left(z^{2^i}\right)$$

Haar filter $G'_0(z)$ leads to a regular 2-D cyclic filter bank with the extended 1-D filter $G_0(z)$.

Unfortunately, it is NOT orthogonal!

Regularity vs. Orthogonality



Haar filter is orthogonal, but results in irregular cyclic filter bank.

Conjecture: Cyclic orthogonal basis construction results in irregularity.

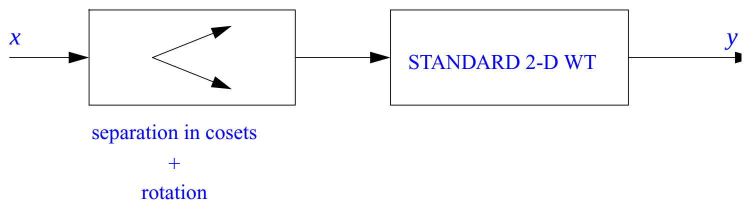
Why? Because decaying in Fourier domain is not ensured in all directions.

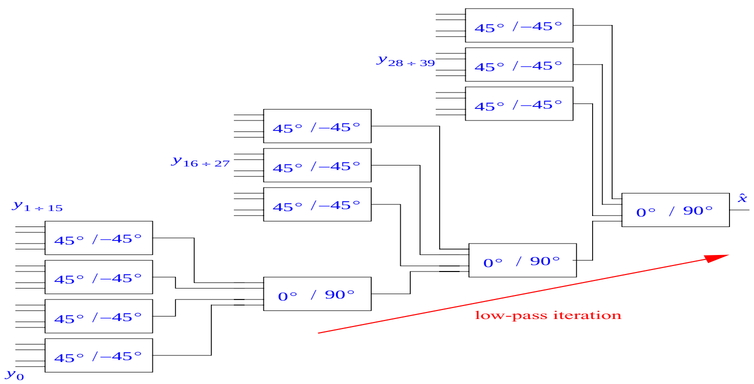
Key problem: Increase in number of cosets.

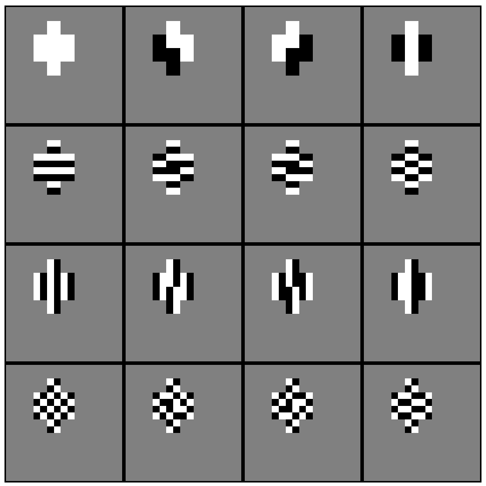
Possible solution: Filtering non-equally distributed across all directions.

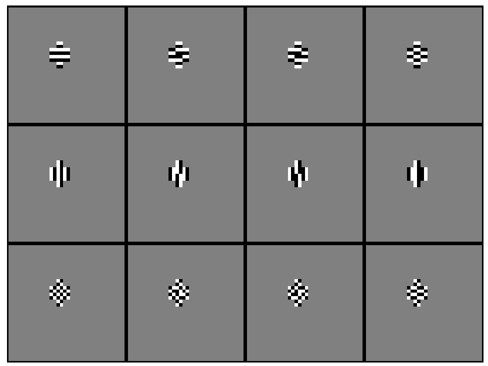
Regularity in 2-Directional Construction

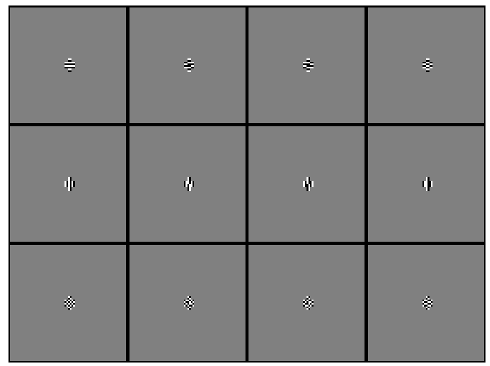
If only two transform directions are iterated, regularity within each coset is inherited from the standard 2-D wavelet transform.











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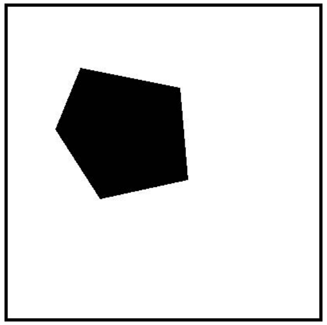
Non-Linear Approximation (NLA)

- Represent an image by a subset of the (largest magnitude) transform coefficients.
- Different transforms can be compared in terms of MSE for the same number of retained coefficients.
- Approximation power for *boundary variation images* (Mallat):
  - The standard 2-D wavelet transform: $O(M^{-1})$
  - Adaptive geometrical processing: not better than $O(M^{-2})$



We first consider a *polygonal image model* - piecewise polynomial images with piecewise linear edges.

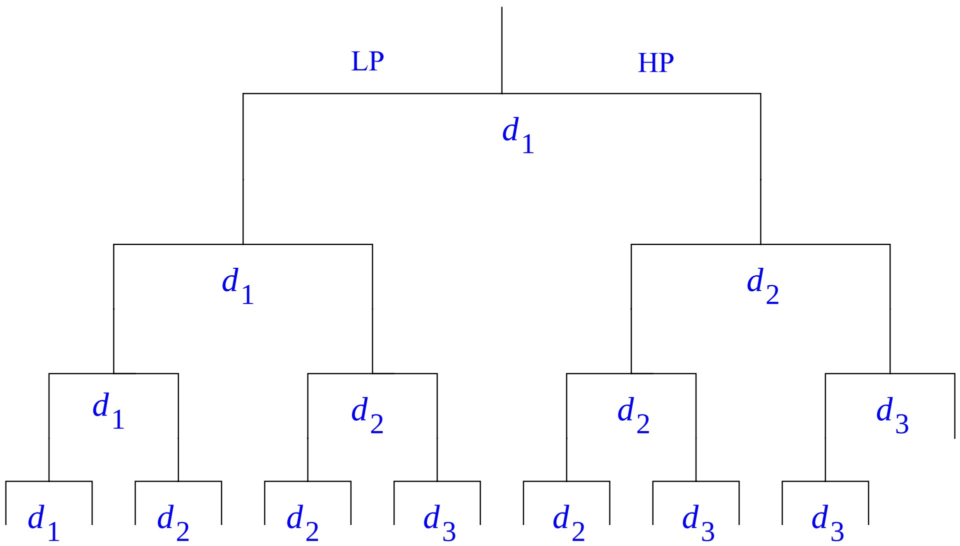
Our approach: Remove directional information by matching the transform directions with edge directions

Theorem: For piecewise polynomial image with piecewise linear edges MDir WT provides sparse representation with $O(2^{-\alpha M})$.

Exponential Approximation Power

The key ideas of the proof of exponential decay of MSE:

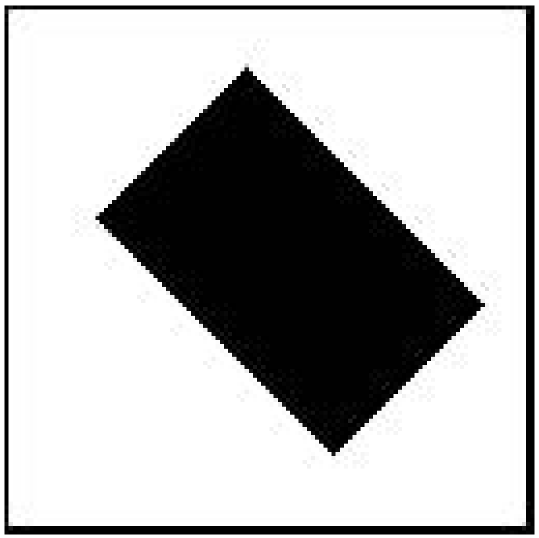
- Magnitudes of the J-level transform coefficients take the values $2^{j/2}$, for $-J \le j \le J$;
- MSE of non-linear approximation is a piecewise linear function with exponentially decaying slope coefficients, 2^j.

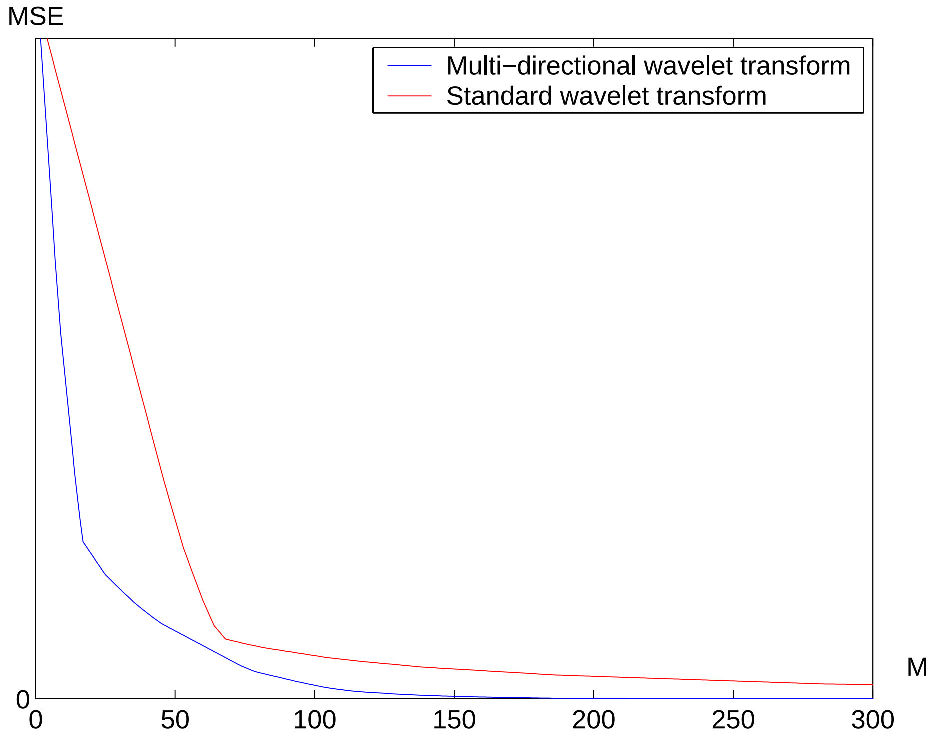


3 directions, 4 levels

The transform is supervised because we need to know dominant directions!

Exponential Approximation Power: Results





Compression

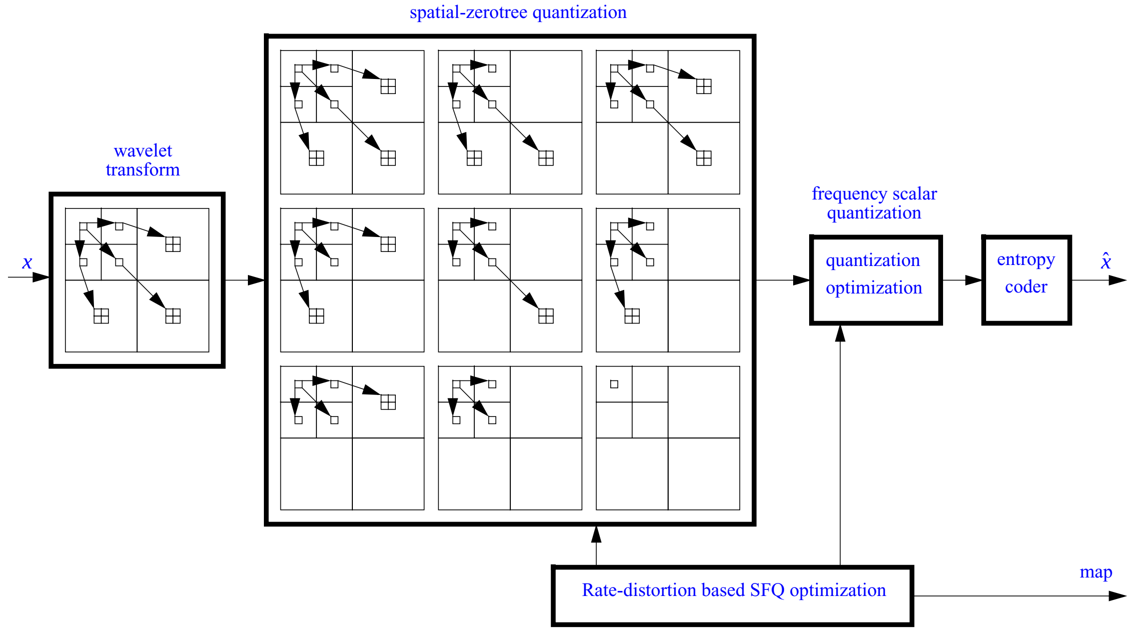
Wavelet-based coders have been applied successfully in image compression:

- Scalar quantization & optimal bit allocation (Antonini et al., 1992)
- Embeeded coding using zerotrees (Shapiro, 1993)
- Set partitioning in hierarchical trees (Said & Pearlman, 1996)
- Space-frequency quantization (Xiong et al., 1998)

- The MDir WT can be incorporated in these coding methods (instead of the standard 2-D WT).
- We propose a modified space-frequency quantization algorithm (MDir SFQ).

Space-Frequency Quantization

SFQ optimizes wavelet-domain zerotree structures in the R-D sense (Z. Xiong).



Optimization in the R-D sense $\longrightarrow$ trade-off between minimization of the rate and distortion using a Lagrangian cost:

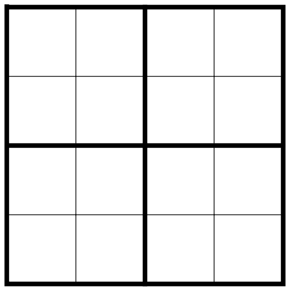
$$\min_q\{J(q) = R(q) + \lambda D(q)\}$$

Prune children if they increase the total cost!

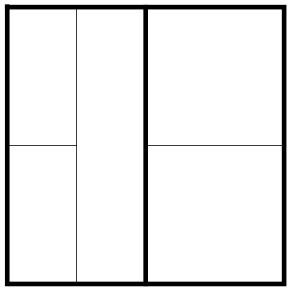
SFQ allows for wavelet packet implementation instead of the standard WT, with slight modifications in the zerotree structures.

Multi-Directional SFQ

To preserve regularity of the basis functions, we use only (any) 2 directions in the transform.



Wavelet Packet



Iteration of single directions

We allow for iteration of a single direction (unlike in WP).

For a specified set of n_d transform directions, there are 4 optimization steps:

1) Choose optimal pair of transform directions – $n_d(n_d - 1)/2$
2) Choose optimal quantization step q – in SFQ: $q = 7.6 + 0.1k$, $k = 1, \ldots 245$
3) Choose optimal frequency-decomposition tree – $2^{2^J - 1}$ (!)
4) Choose optimal zerotree structure – SFQ

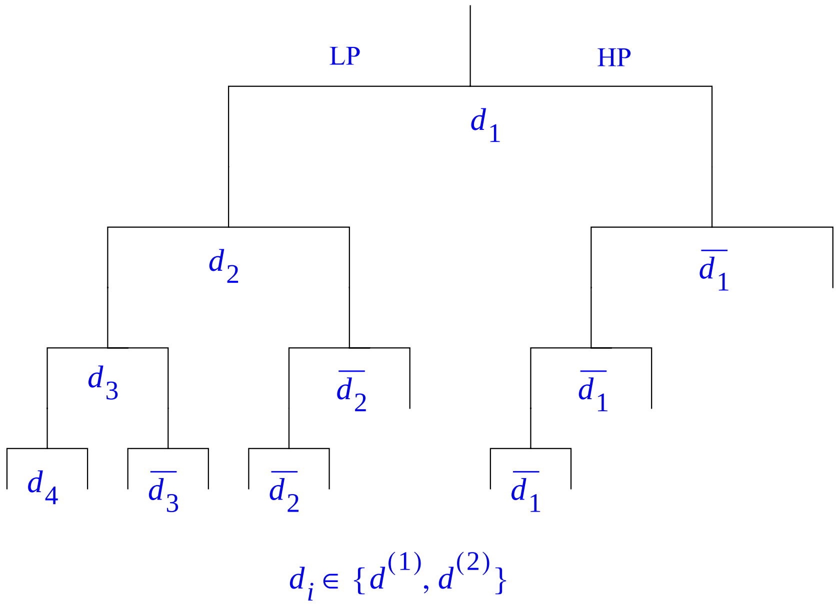
Lagrangian parameter λ is chosen depending on the total bit budget.

Multi-Directional SFQ

How to reduce complexity?

1) Separate SFQ from the other optimization steps. For the others, estimate rate as first-order entropy and distortion as scalar quantization MSE.

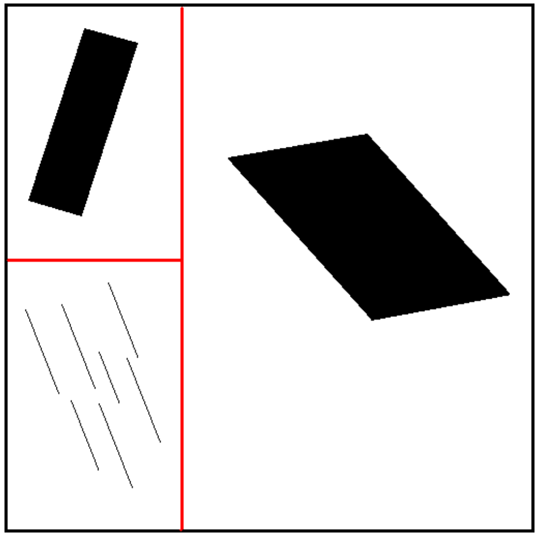
2) Implement growing frequency decomposition tree optimization instead of pruning full trees.



- Transform directions are not repeated in the high-pass branches.
- Number of possible trees is now 2^J.
- We continue growing-tree procedure in the terminate nodes.
- This is done for each pair of transform directions and each q.

MDir SFQ: Spatial Segmentation

Directionality is local feature



- Apply optimization algorithm on the whole image
- Segment in quad-tree way and repeat optimization for each segment
- Compare the costs of segments and image and keep better
- Repeat iteratively until the minimal size of segments is reached

For each segment we choose optimal:

1) pair of transform directions,
2) quantization step q,
3) frequency-decomposition tree,
4) zerotrees.

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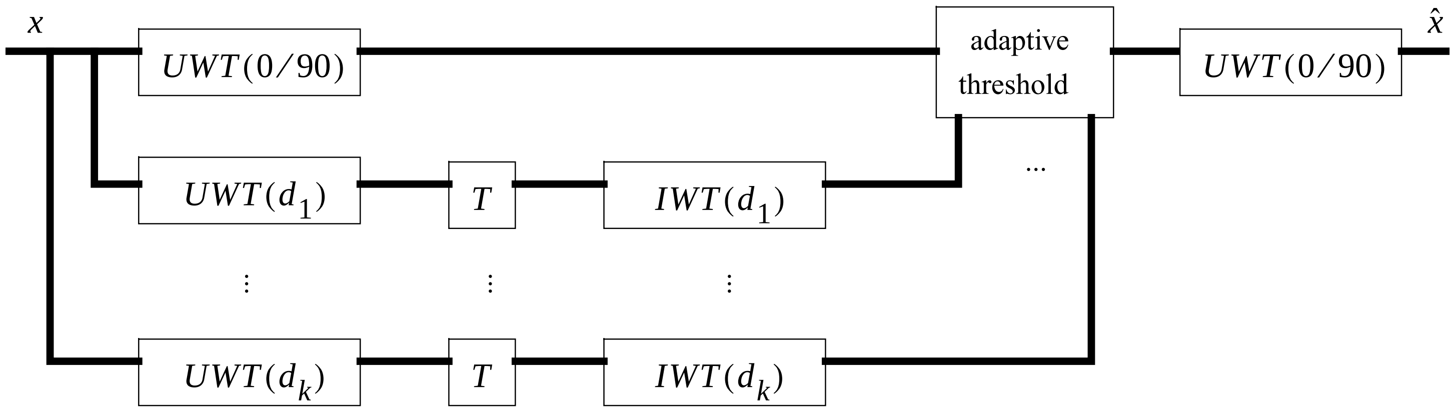
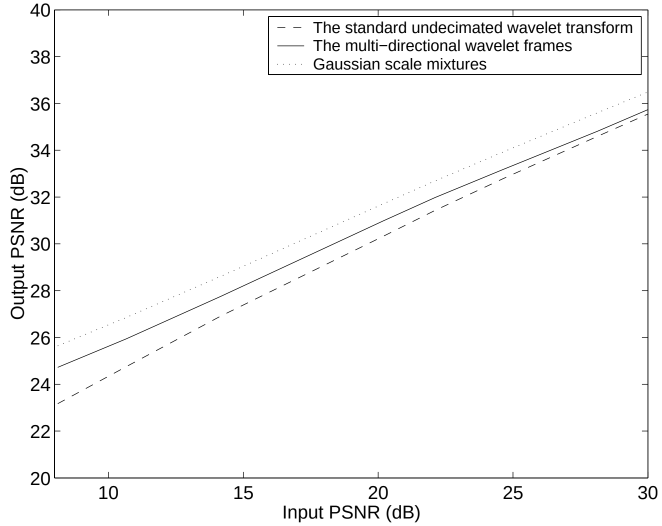
Denoising

- Redundant undecimated MDir WT is used. There is NO directional interaction!
- Thresholding in the transform domain (Donoho & Johnstone, 1994).

First denoising approach: We use two thresholding steps and k transform directions



Denoising Results



original

10.66dB

2-D UWT

24.75dB

MDir UWT

25.94dB

Denoising Results: Details

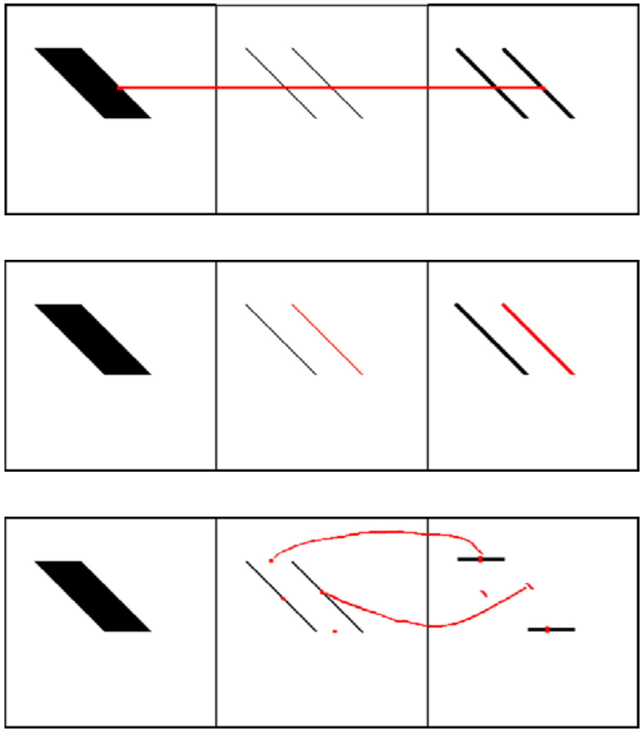
Standard 2-D UWT

MDir UWT

Future Improvements in Denoising

The goal is to enforce three types of coherence in images:

- multi-scale
- geometrical (spatial)
- angular



Conclusion

- The MDir WT retains simplicity of the standard WT, but allows for capturing directional features in images and provides a sparse representation
- The MDir WT constructed using two transform directions ensures regular basis functions
- Non-linear approximation using the MDir WT with transform directions matched with dominant directions in images achieves exponential decay of mean-square error.
- MDir WT can be incorporated in the wavelet-based compression alogithms, like SFQ
- Simple denoising algorithm achieves promising results. Further improvements can be obtained by enforcing coherences in images across scale, space and directions